

Exploring the Phase Structure and the Dynamics of QCD

RIKEN Lunch Seminar
12/04/14 BNL



Nils Strodthoff, ITP Heidelberg

Outline

QCD Phase Structure

- QCD phase structure from functional approaches
- Quenched QCD in the vacuum

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Dynamics I Spectral functions

- Spectral functions from a Euclidean framework
- Mesonic spectral functions in simple models

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Dynamics II Transport Coefficients

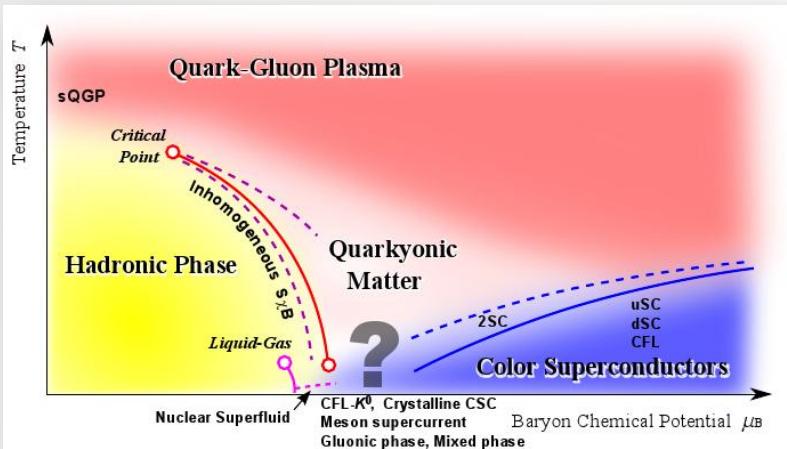
- Kubo formula with expansion in full propagators/vertices
- Transport coefficients in YM and QCD

QCD Phase Structure

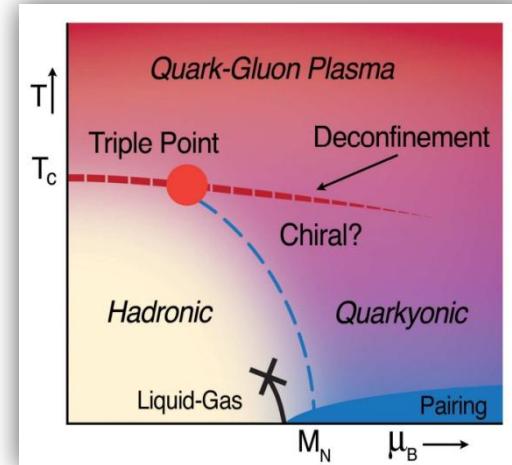
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- Mitter, Pawłowski, NSt arXiv:1411.7978

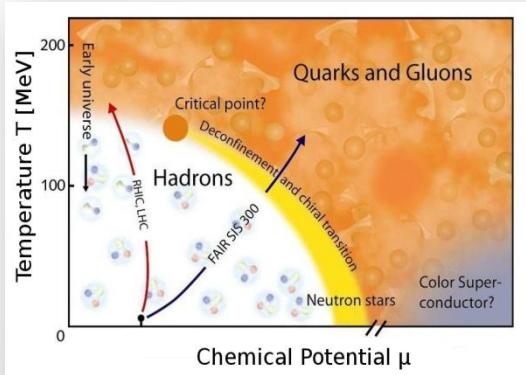
The QCD phase diagram?



➤ Fukushima, Hatsuda Rept.Prog.Phys. **74** (2011) 014001

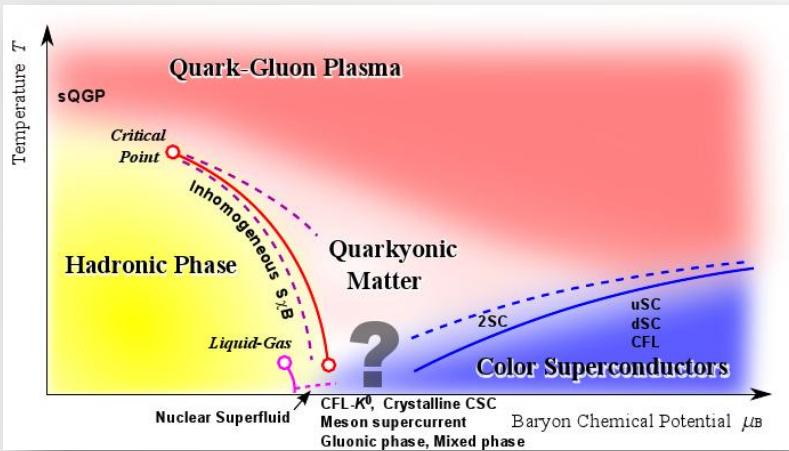


➤ A. Andronic et al Nucl.Phys. **A837** (2010)

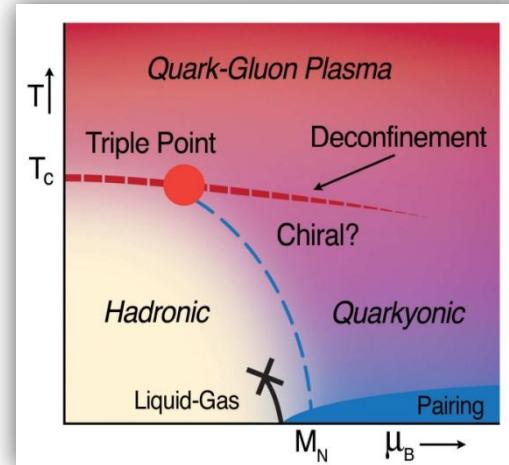


➤ adapted from GSI

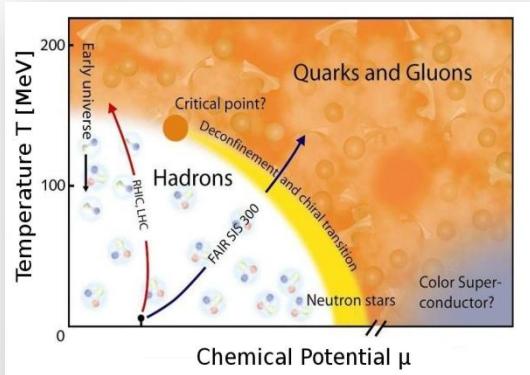
The QCD phase diagram?



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Phase structure at large chemical potentials largely unknown due to sign problem in lattice QCD...

➤ adapted from GSI

Continuum perspective

...using functional approaches

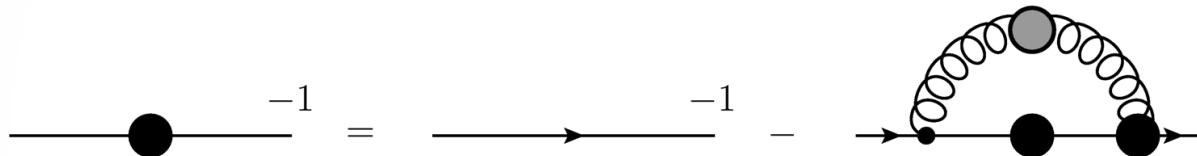
Functional relations between off-shell Green's functions

Continuum perspective

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e.g. Dyson-Schwinger equation for quark propagator



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Functional relations between off-shell Green's functions

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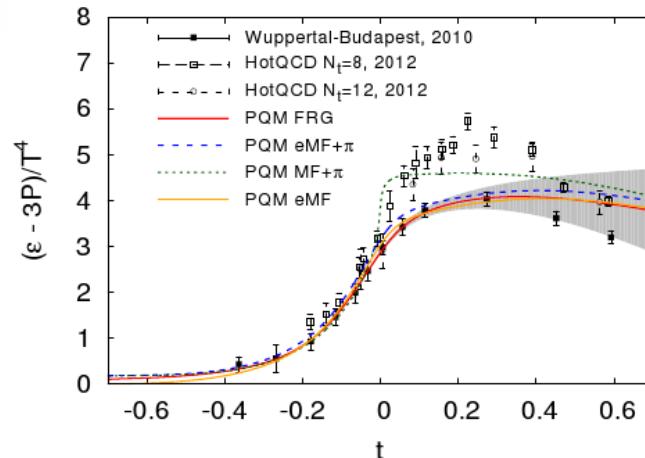
$$\text{---} \bullet^{-1} = \text{---} \rightarrow^{-1} - \text{---} \rightarrow \text{---} \bullet$$

- ✓ Easy access to mechanisms:
 - Chiral symmetry breaking
 - Confinement

- ✓ Complementary to the lattice
- ✓ No sign problem
- ✓ Effective models incorporated

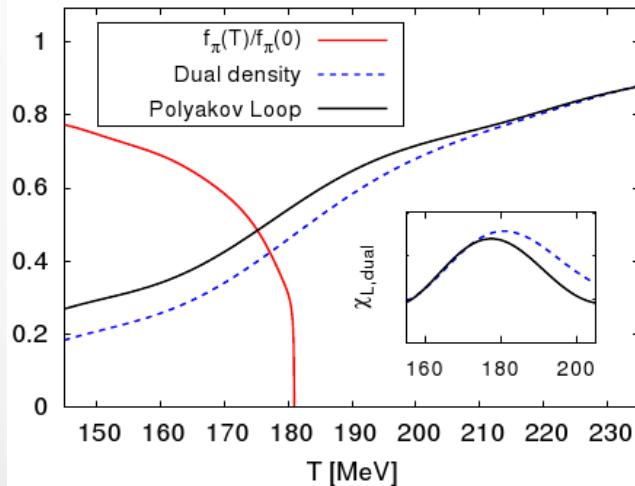
Functional Approaches: finite T

PQM model, Nf=2+1, FRG



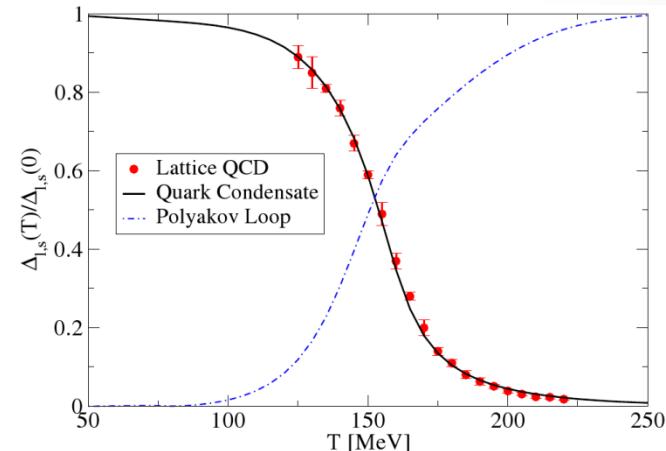
➤ Herbst, Mitter, Pawłowski, Schaefer, Stiele
Phys.Lett. **B731** (2014) 248-256

Matter+Glue system, Nf=2, FRG



➤ Braun, Haas, Marhauser, Pawłowski
PRL **106** (2011) 022002

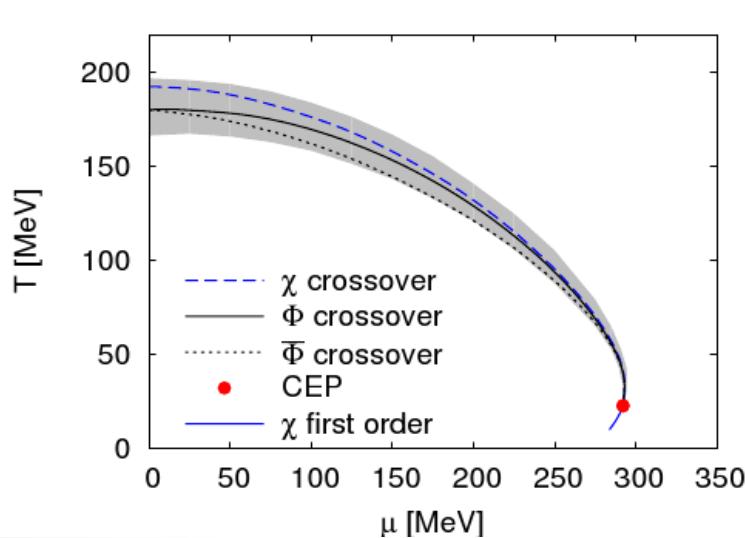
Quark propagator, Nf=2+1, DSE



➤ Fischer, Luecker, Welzbacher
Phys.Rev. **D90** (2014) 034022

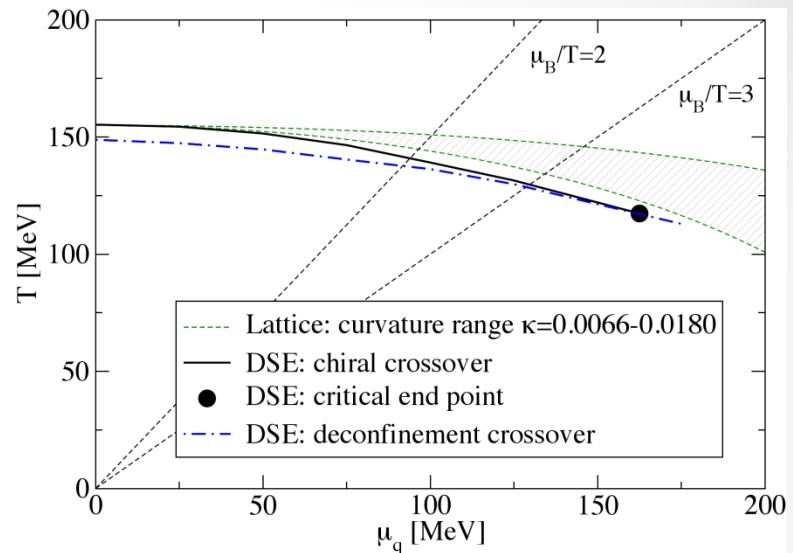
Functional Approaches: finite T & μ

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- Herbst, Pawlowski, Schaefer Phys.Lett. **B696** (2011)

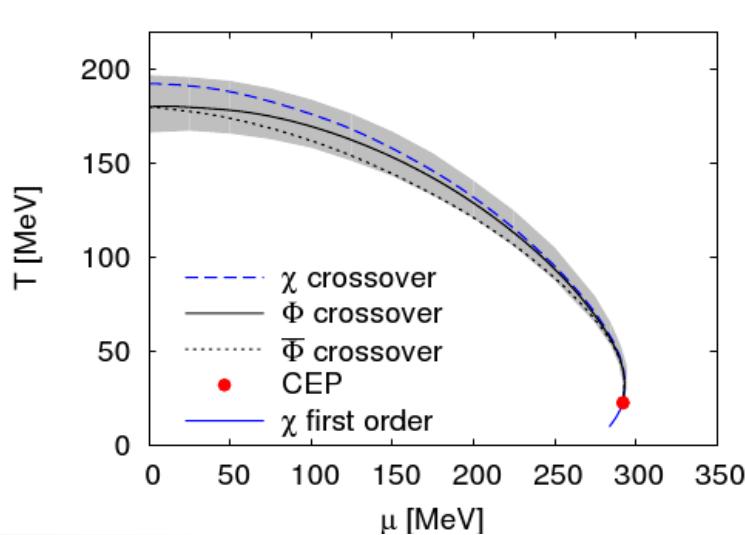
Quark propagator, Nf=2+1, DSE



- Fischer, Fister, Luecker, Pawlowski Phys.Lett. **B732** (2014) 273-277

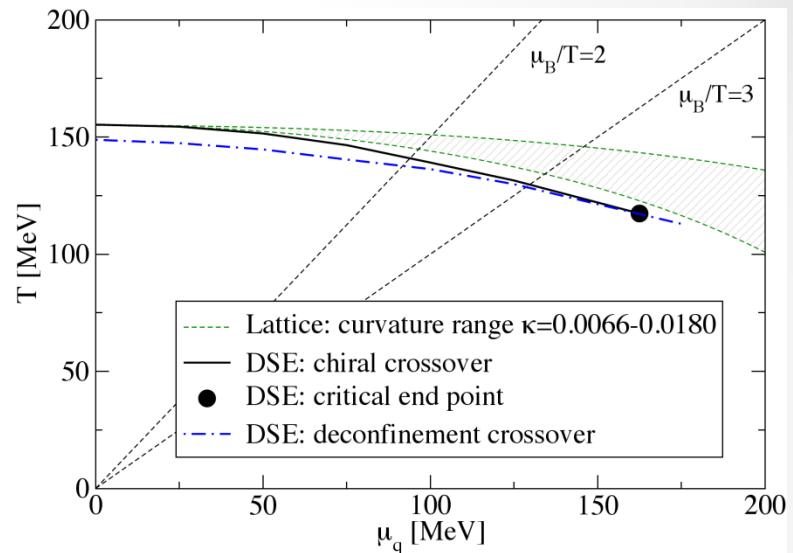
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But: so far all **require additional phenomenological input**

PQM-model: UV parameters, glue input (Polyakov-loop potential)

DSE calculation: vertex models e.g. for the quark-gluon vertex

fQCD Collaboration

fQCD Collaboration (J.Braun, L.Fister, T.K. Herbst, M.Mitter,
J.M. Pawłowski, F. Rennecke and N. Strodthoff)

- Mitter, Pawłowski, NSt arXiv:1411.7978
- Braun, Fister, Pawłowski, Rennecke arXiv:1412.1045

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- ✓ Finite μ requires fluctuations to be **quantitatively** under control
 - ✓ Mismatches in fluctuation scales lead to large systematic errors at finite μ
 - Helmboldt, Pawłowski, NSt arXiv:1409.8414

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- ✓ No phenomenological input (vertex models, running couplings...)
- ✓ Input parameters only the fundamental parameters of QCD

α_s (20 GeV) **strong running coupling**
 M_q (20 GeV) \approx 1-2 MeV **current quark mass**

at large perturbative momenta.

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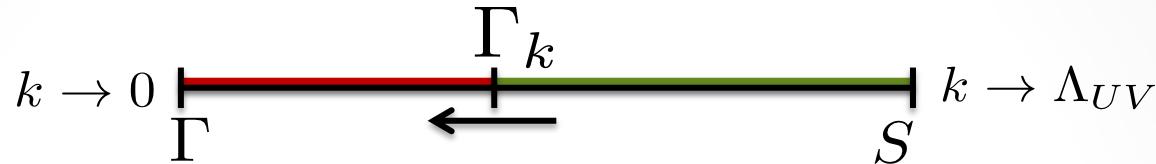
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at large perturbative momenta.

- ✓ Quantitative FRG approach towards the investigation of the phase diagram and the hadron spectrum

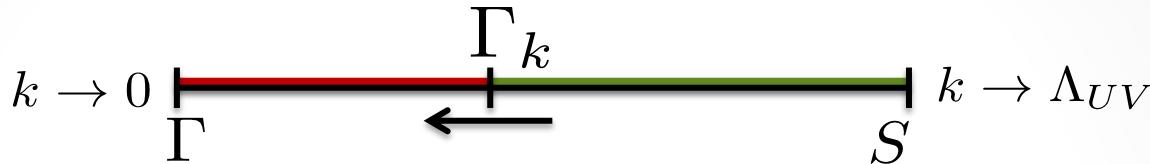
Functional RG for QCD

- Spirit of **Wilson RG**: Calculate full quantum effective action Γ by integrating fluctuations with momentum k



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Functional Renormalization Group (FRG)

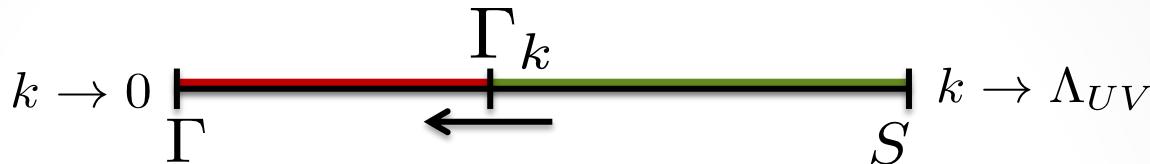
$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{ (gluon loop)} - \text{ (ghost loop)} - \text{ (quark loop)} + \frac{1}{2} \text{ (hadrons loop)}$$

Free energy/
Grand potential

gluon ghost quark hadrons

Functional RG for QCD

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Functional Renormalization Group (FRG)

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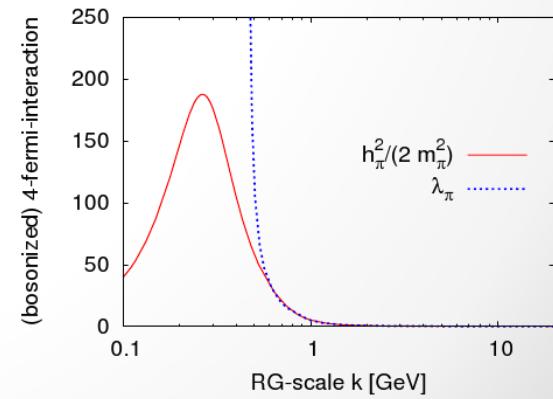
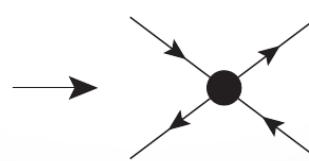
gluon ghost quark hadrons



Dynamical hadronization

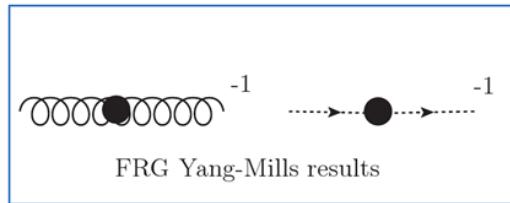
➤ Gies, Wetterich Phys.Rev. D65 (2002) 065001

Store resonant 4-Fermi structures in terms of
effective mesonic interactions



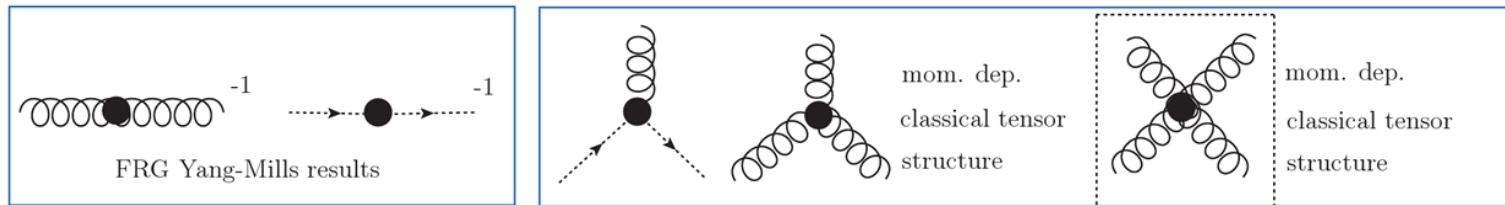
Truncation

Vertex expansion



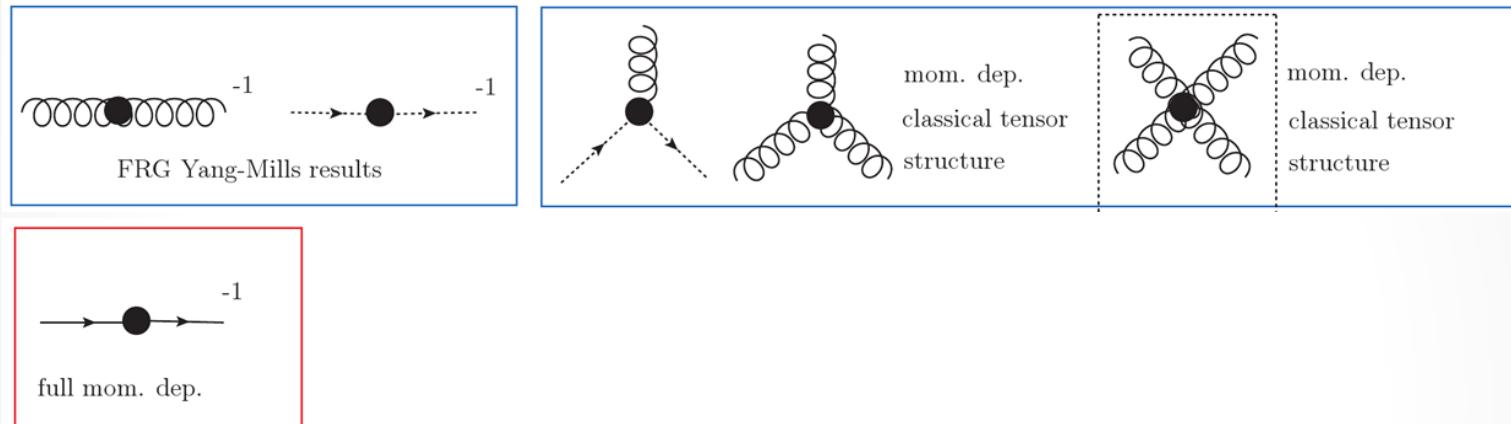
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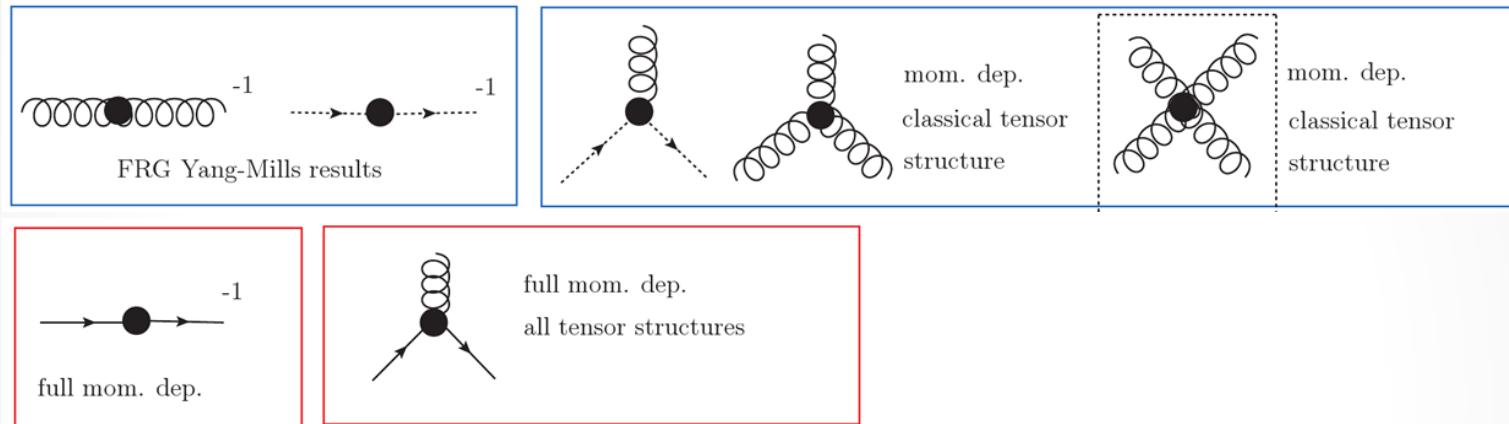
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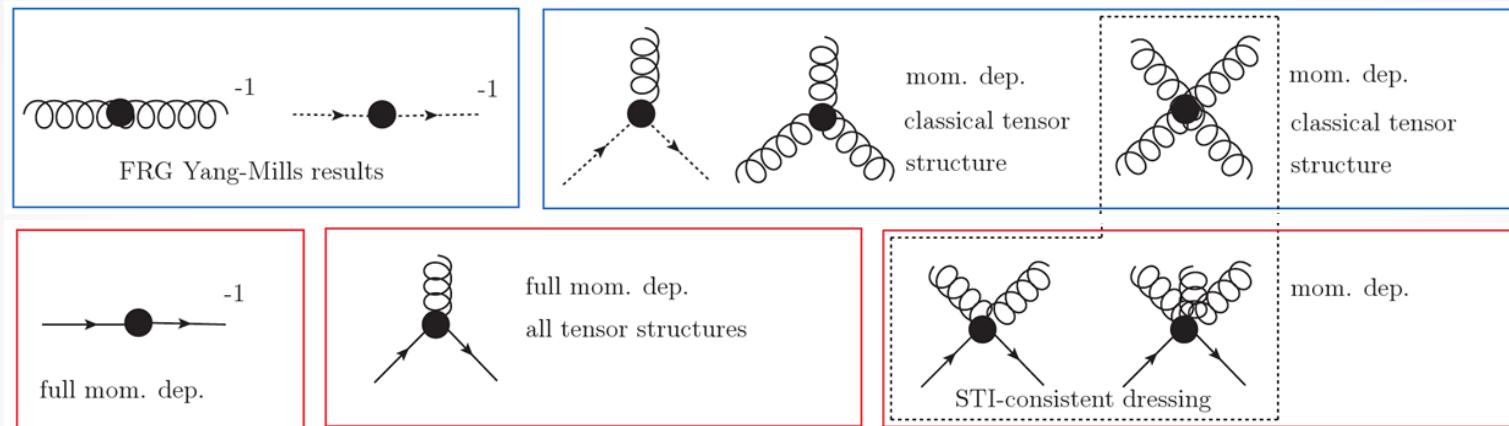
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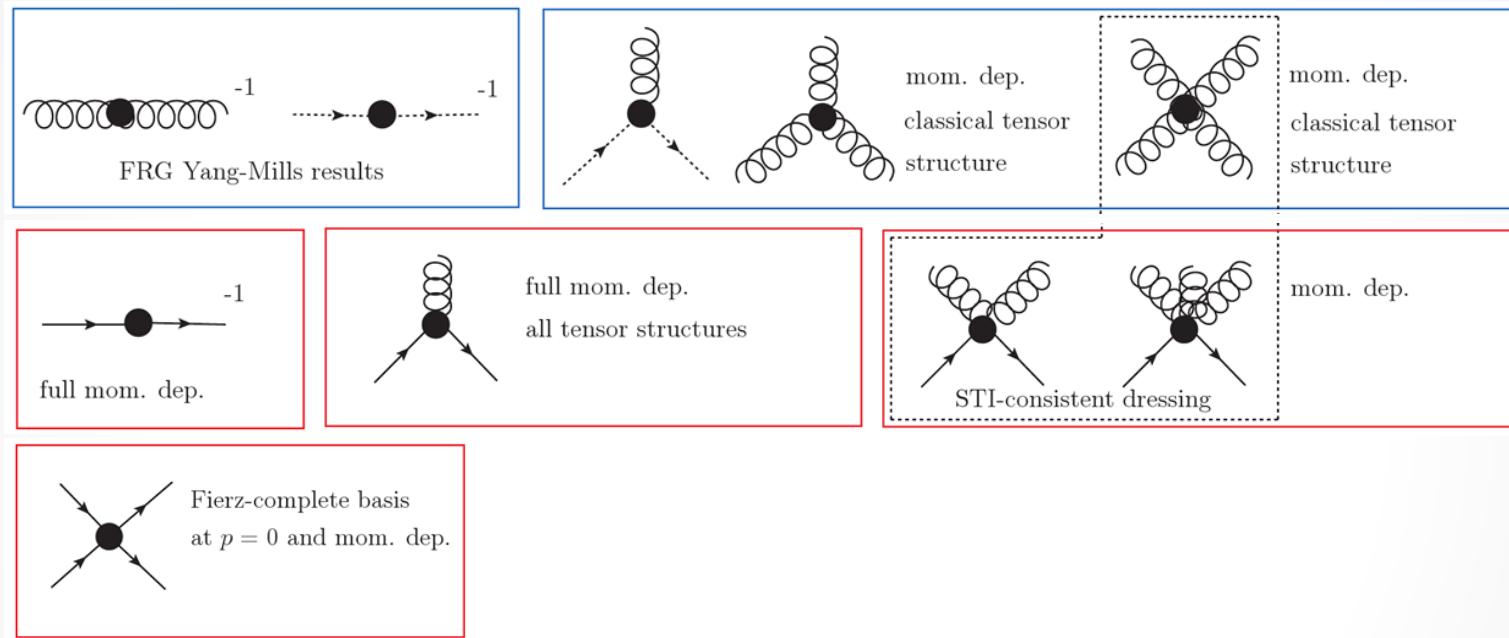
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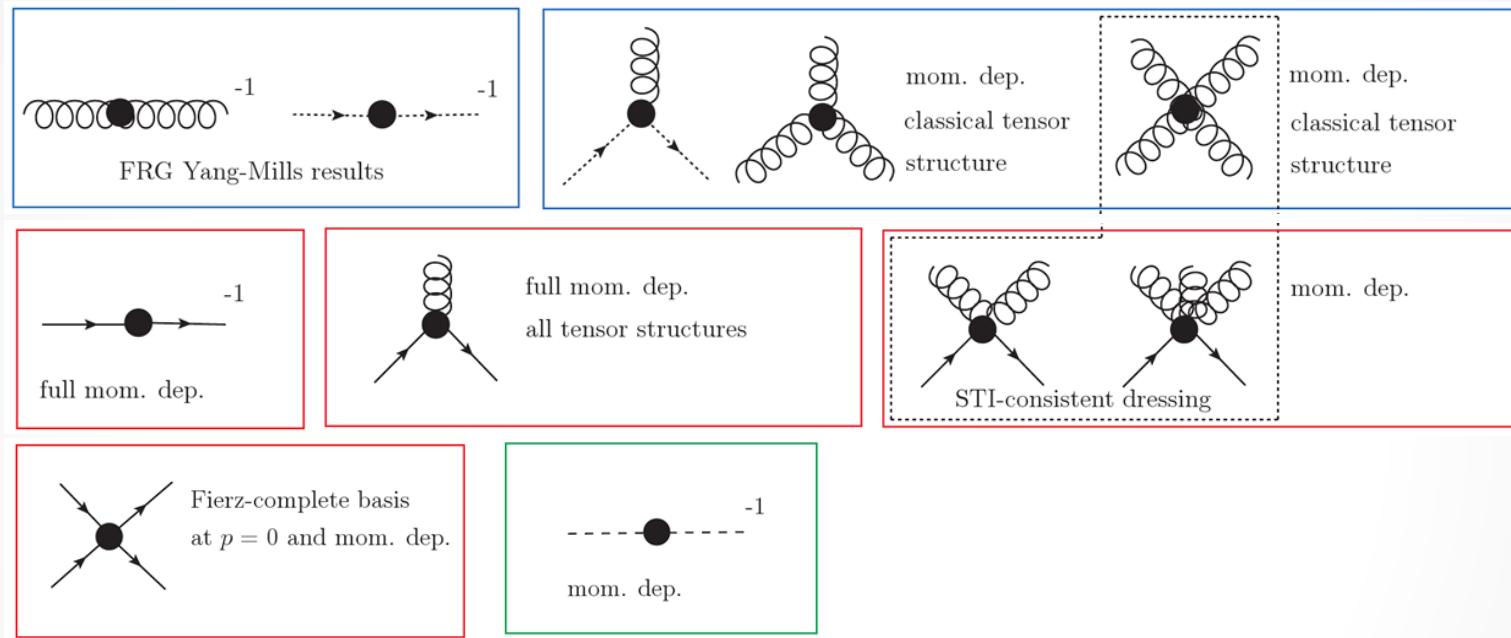
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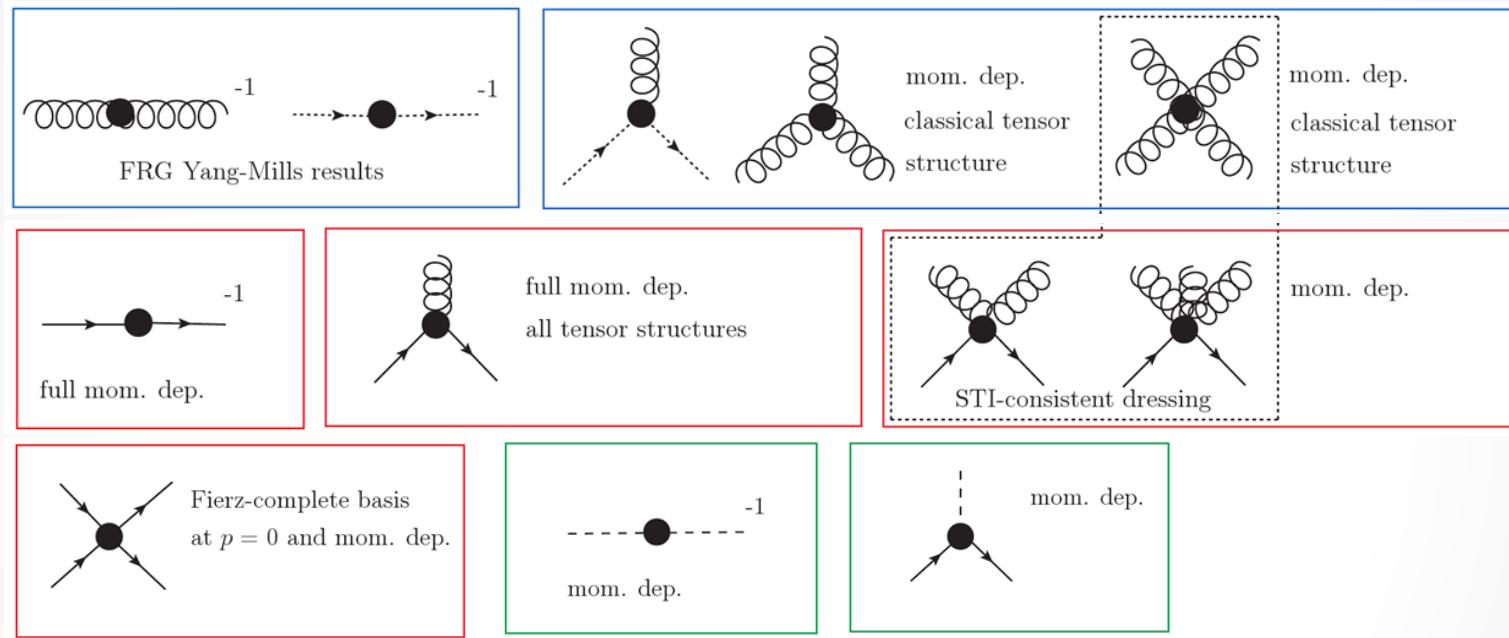
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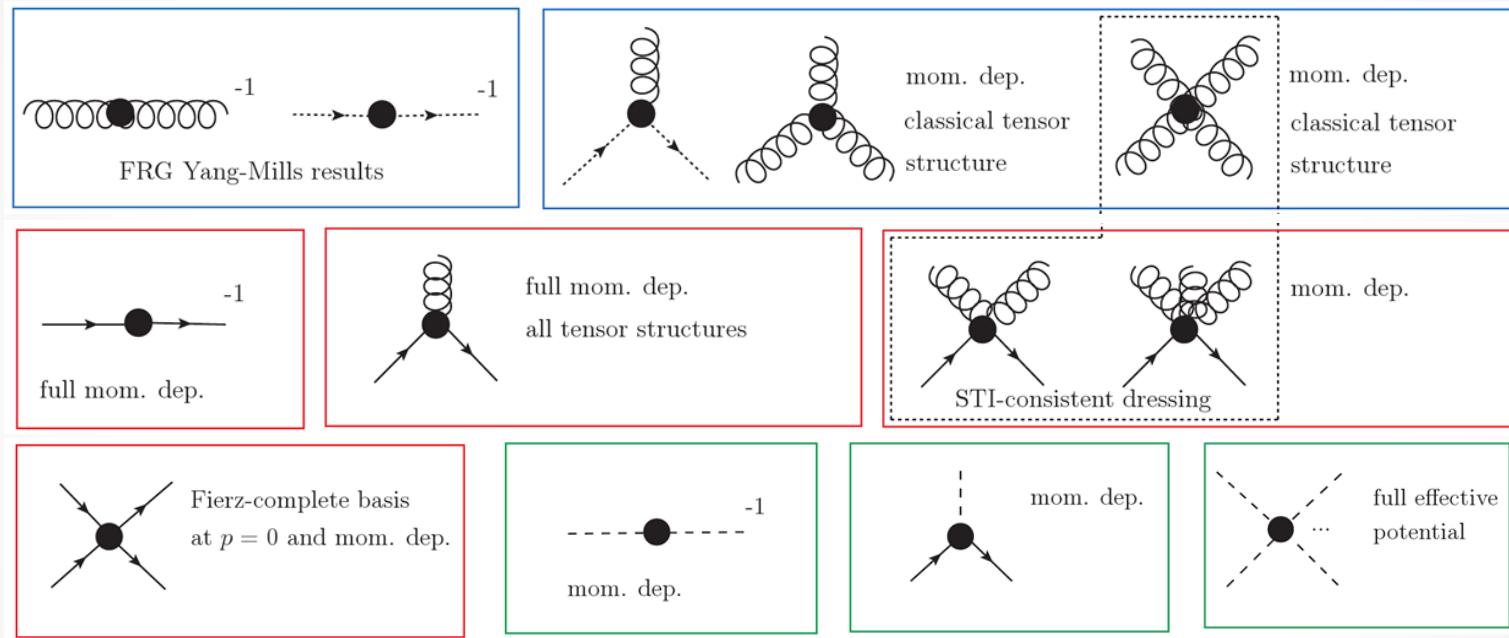
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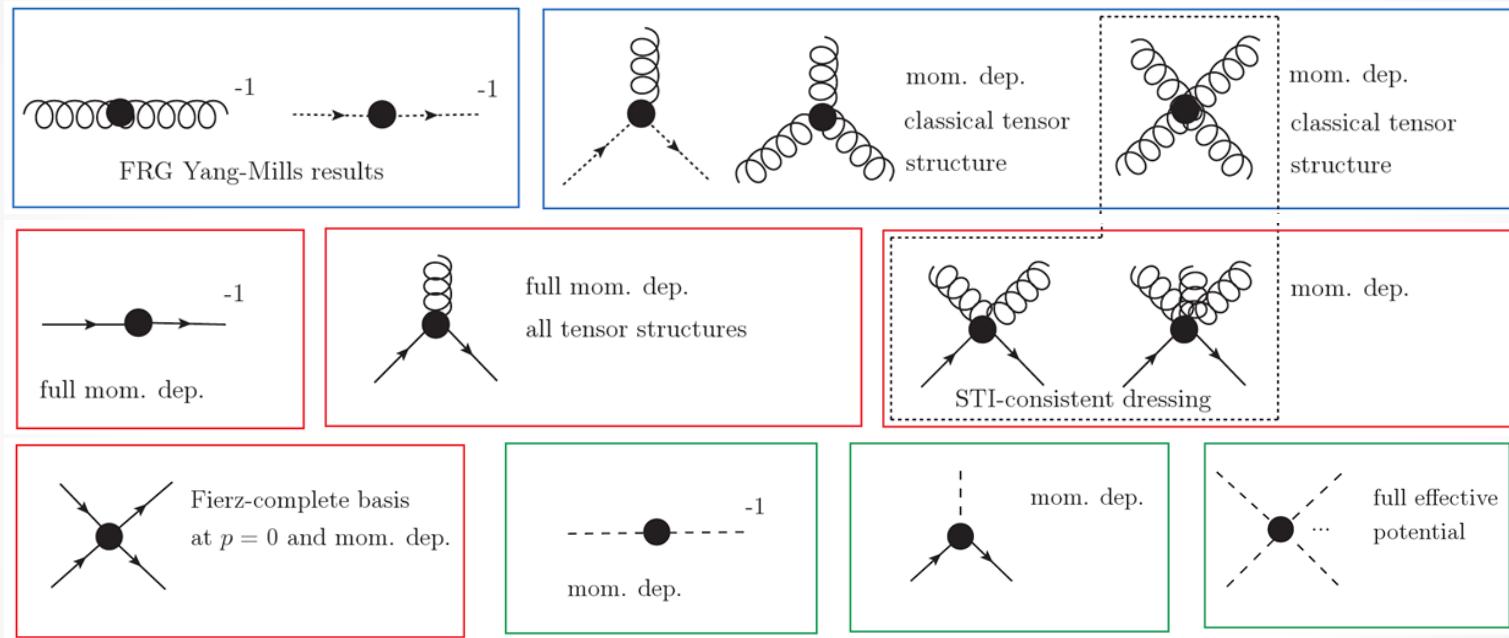
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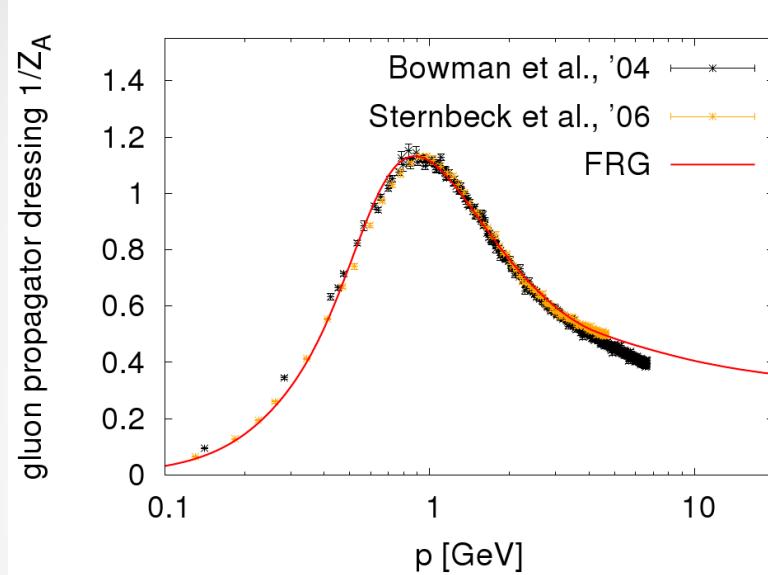


Computer-algebraic generation of equations using DoFun

➤ Huber, Braun Comput.Phys.Commun. **183** (2012) 1290-1320

Propagators ($T=0$)

Quenched gluon propagator (input)



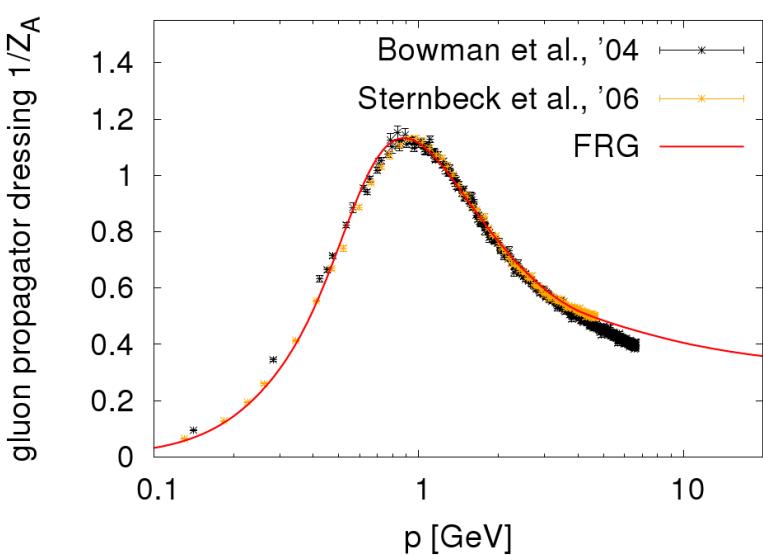
$$\Gamma_{A^2}^{\mu\nu}(p) = Z_A(p)p^2\Pi_T^{\mu\nu}(p)$$

- Bowman et al Phys.Rev. **D70**, 034509 (2004)
- Sternbeck et al PoS **LAT2006**, 076 (2006)
- Fischer, Maas, Pawłowski Annals Phys. **324**, 2408 (2009)
- Fister, Pawłowski in prep.

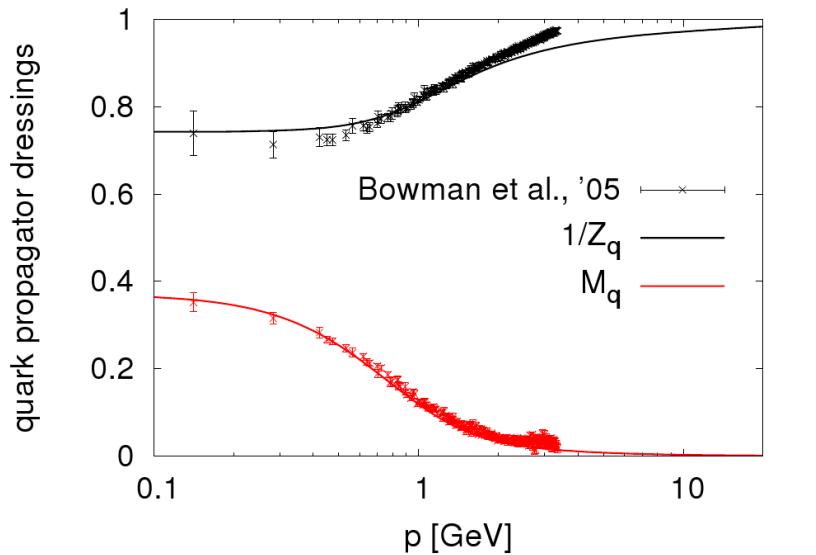


Propagators ($T=0$)

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Quark propagator



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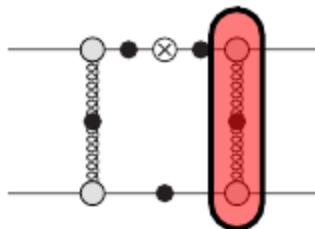
$$\Gamma_{\bar{q}q}(p) = Z_q(p)(i\gamma + M_q(p))$$

Very good agreement with (quenched) lattice results!

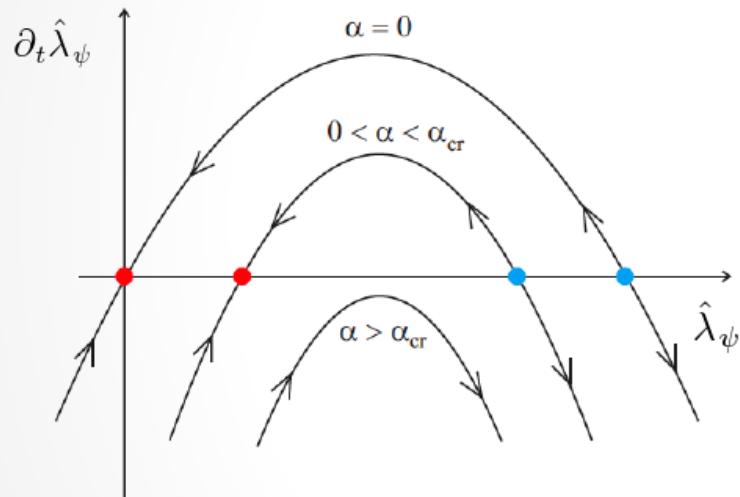
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- Mitter, Pawłowski, NSt arXiv:1411.7978

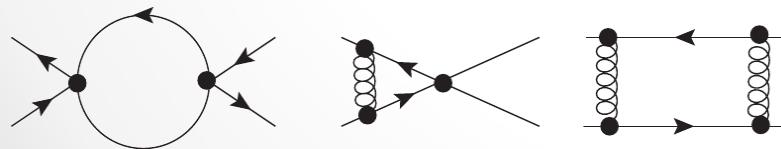
Chiral symmetry breaking



β -function:



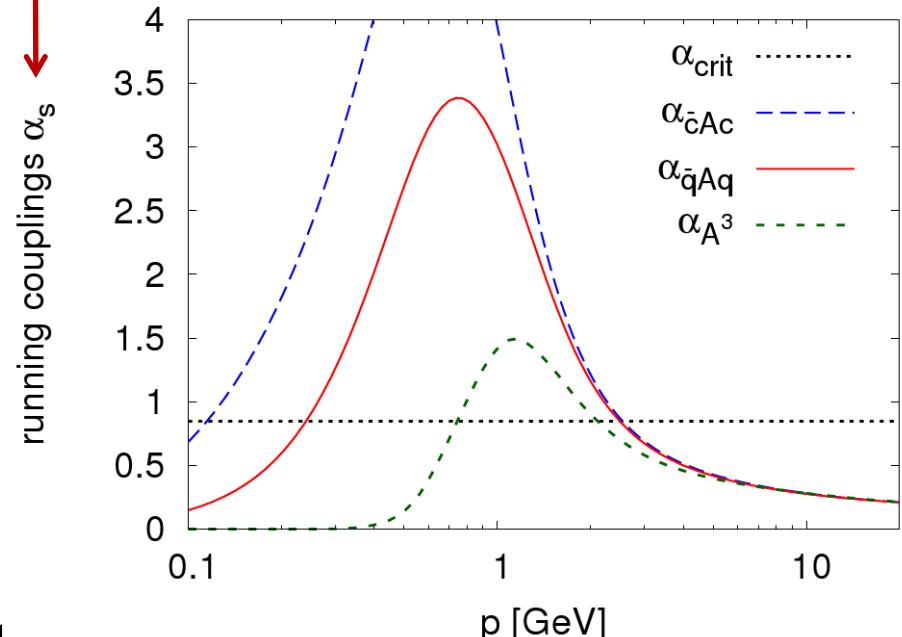
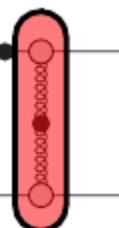
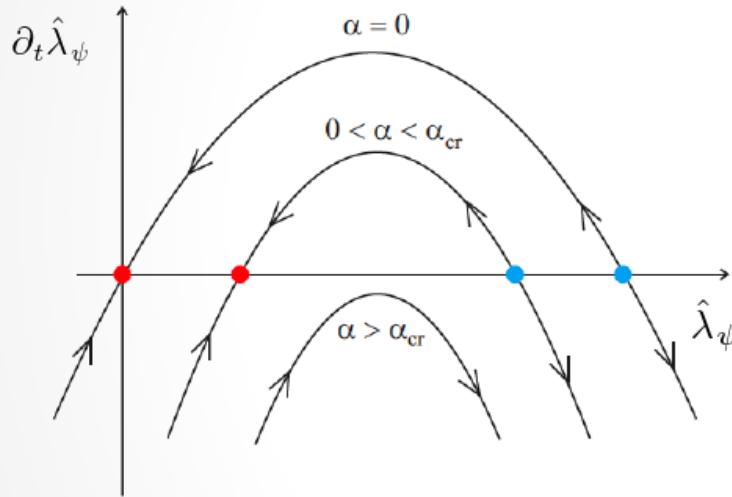
$$k \partial_k \hat{\lambda}_\psi = (d - 2) \hat{\lambda}_\psi - a \hat{\lambda}_\psi^2 - b \hat{\lambda}_\psi g^2 - c g^4$$



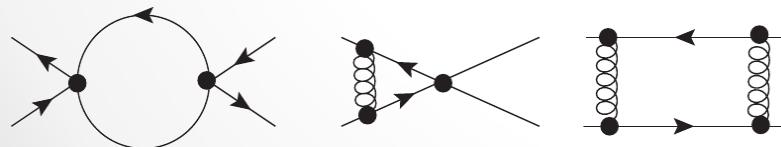
➤ Review: Braun J.Phys. **G39** (2012) 033001

Chiral symmetry breaking

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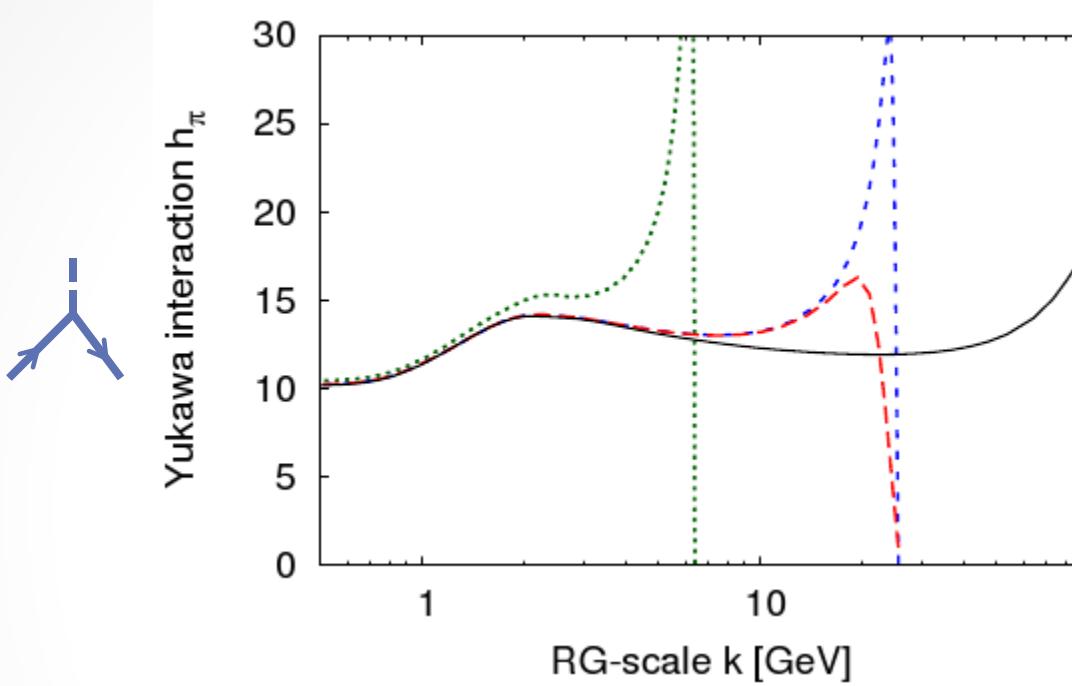
$$k \partial_k \hat{\lambda}_\psi = (d - 2) \hat{\lambda}_\psi - a \hat{\lambda}_\psi^2 - b \hat{\lambda}_\psi g^2 - c g^4$$



- reflects gluon mass gap
- area above the critical value decides

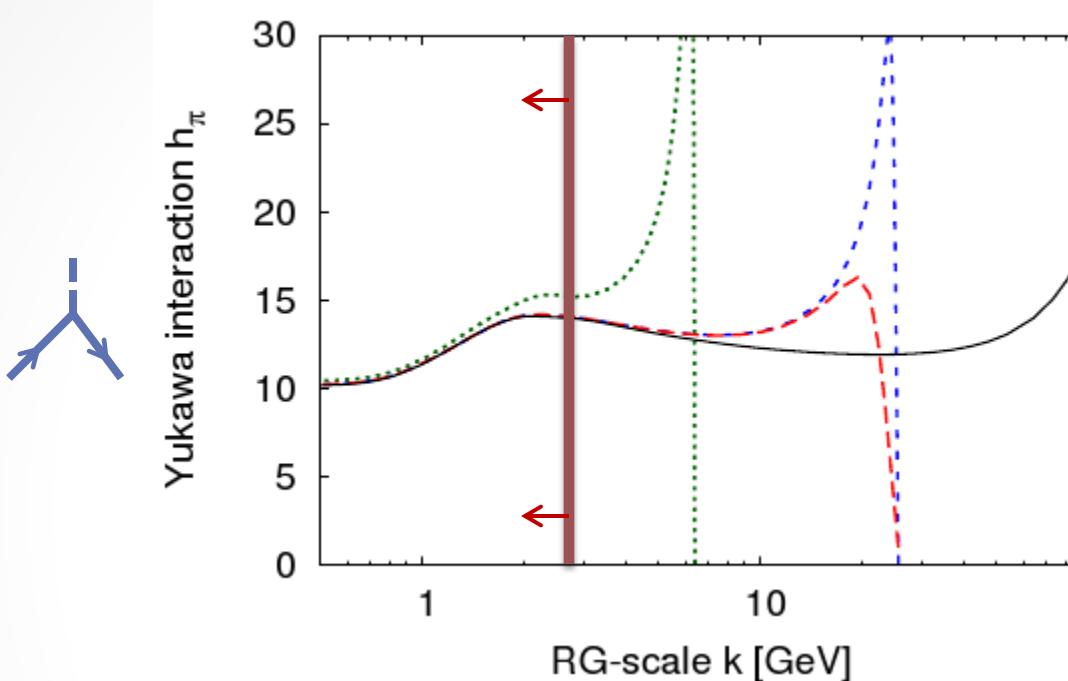
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Effective model perspective



- Independence of initial scale and initial condition

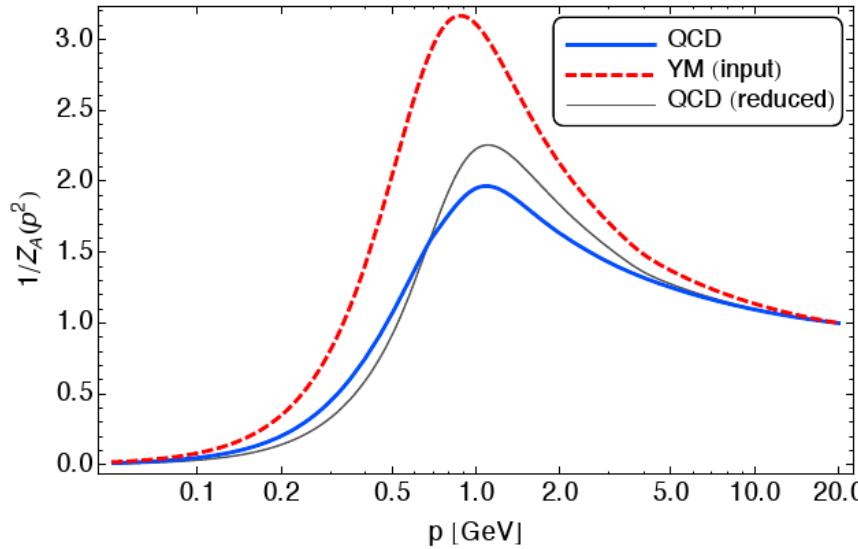
Effective model perspective



- Independence of initial scale and initial condition
- only requirement: decoupling of gluons
- **Low-energy models completely fixed by QCD flow**

Outlook

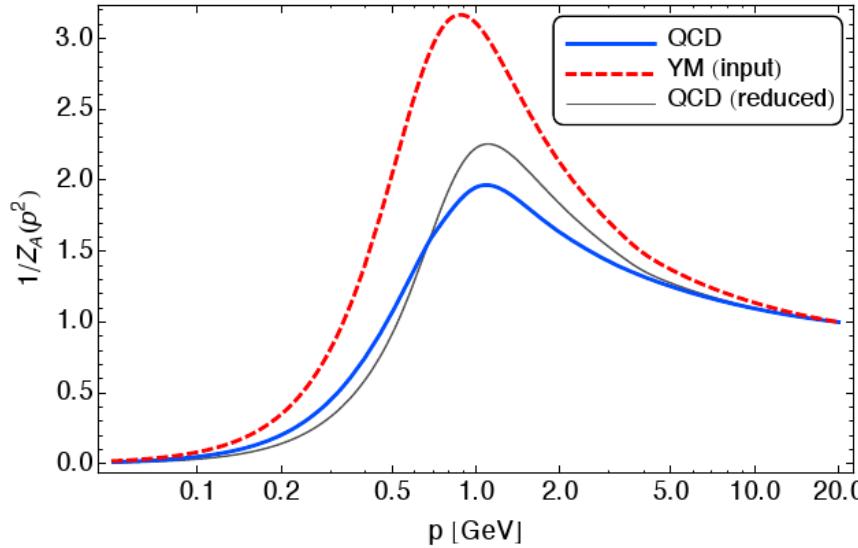
Unquenching: first qualitative results available



- Braun, Fister, Pawlowski, Rennecke arXiv:1412.1045

Outlook

Unquenching: first qualitative results available



➤ Braun, Fister, Pawlowski, Rennecke arXiv:1412.1045

Shopping list

- ✓ Quantitative results in the vacuum
- Full unquenching
- Quantitative investigations in the vacuum (YM vertices, 4-Fermi)
- Transition to low-energy effective models
- Finite temperature
- Finite Density (important: role of baryonic/diquark d.o.f.)

Dynamics I

Spectral Functions

• • •

- Kamikado, NSt, von Smekal, Wambach Eur.Phys.J. **C74** (2014) 2806
- Tripolt, NSt, von Smekal, Wambach Phys.Rev. **D89** (2014) 034010
- Helmboldt, Pawłowski, NSt arXiv:1409.8414

Spectral Functions

Real-time observable from Euclidean framework

$$\Gamma_R^{(2)}(\omega, \vec{p}) = -\lim_{\epsilon \rightarrow 0} \Gamma_E^{(2)}(-i(\omega + i\epsilon), \vec{p})$$

$$\rho(\omega, \vec{p}) = \frac{\text{Im } \Gamma_R^{(2)}(\omega, \vec{p})}{\text{Im } \Gamma_R^{(2)}(\omega, \vec{p})^2 + \text{Re } \Gamma_R^{(2)}(\omega, \vec{p})^2}$$

requires analytical continuation from Euclidean to Minkowski signature
numerically hard or even ill-posed problem

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Popular approaches (based on Euclidean data)

- Maximum Entropy Method (MEM)
- Padé Approximants

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Alternative: analytic continuation on the level of the functional equation

- Kamikado, NSt, von Smekal, Wambach Eur.Phys.J. **C74** (2014) 2806
- Floerchinger JHEP 1205 (2012) 021
- Strauss, Fischer, Kellermann PRL **109** (2012) 252001

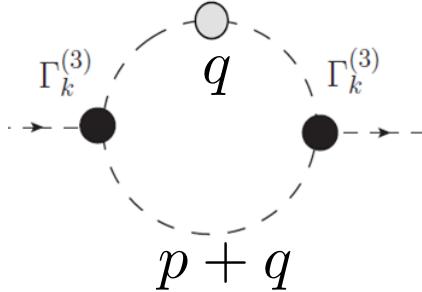
Analytical continuation

➤ Kamikado, NSt, von Smekal, Wambach Eur.Phys.J. **C74** (2014) 2806

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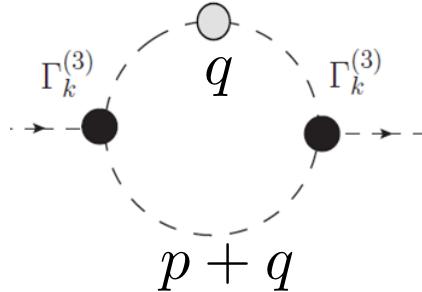
- Compute flow equation for Euclidean 2-point function
perform analytically for 3d regulator function $R = \vec{p}^2 r(\vec{p}^2)$



Analytical continuation

➤ Kamikado, NSt, von Smekal, Wambach Eur.Phys.J. C74 (2014) 2806

- Compute flow equation for Euclidean 2-point function
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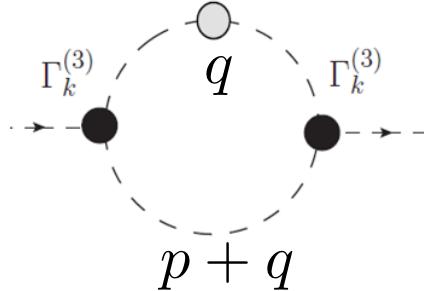
- Perform analytical continuation in ext. momentum

$$p_0 \rightarrow -i(\omega + i\epsilon)$$

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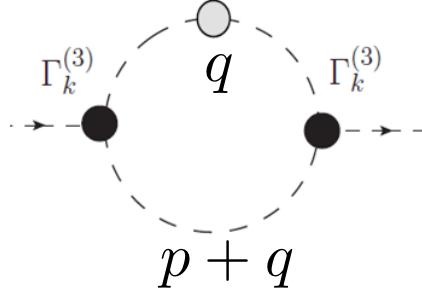
- Ensure correct continuation

$$n_{B/F}(E + ip_0) \rightarrow n_{B/F}(E)$$

Analytical continuation

➤ Kamikado, NSt, von Smekal, Wambach Eur.Phys.J. C74 (2014) 2806

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perform analytically for 3d regulator function $R = \vec{p}^2 r(\vec{p}^2)$



- Perform analytical continuation in ext. momentum

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- Ensure correct continuation

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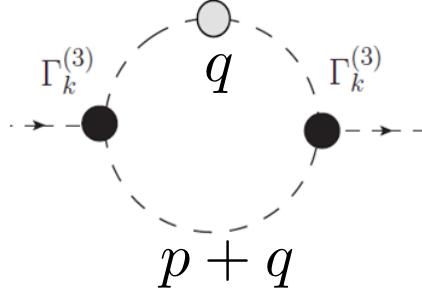
- For small but finite ϵ compute real and imaginary part of

$$-\Gamma_E^{(2)}(-i(\omega + i\epsilon), \vec{p})$$

Analytical continuation

➤ Kamikado, NSt, von Smekal, Wambach Eur.Phys.J. C74 (2014) 2806

- Compute flow equation for Euclidean 2-point function
perform analytically for 3d regulator function $R = \vec{p}^2 r(\vec{p}^2)$



- Perform analytical continuation in ext. momentum

$$p_0 \rightarrow -i(\omega + i\epsilon)$$

- Ensure correct continuation

$$n_{B/F}(E + ip_0) \rightarrow n_{B/F}(E)$$

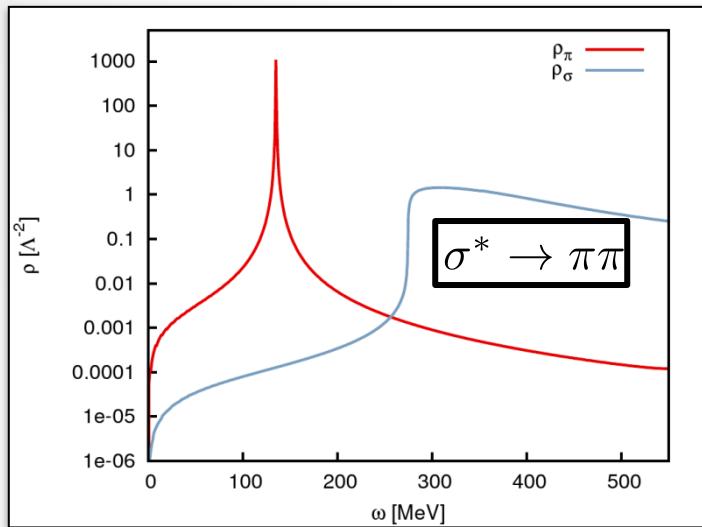
- For small but finite ϵ compute real and imaginary part of

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Test cases: simple bosonic/ Yukawa models

Mesonic Spectral Functions

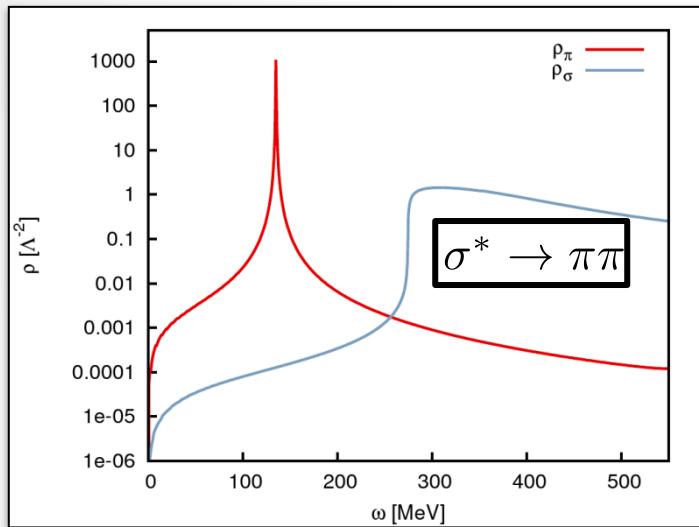
O(N) model (T=0)



- Kamikado, NSt, von Smekal, Wambach
Eur.Phys.J. **C74** (2014) 2806

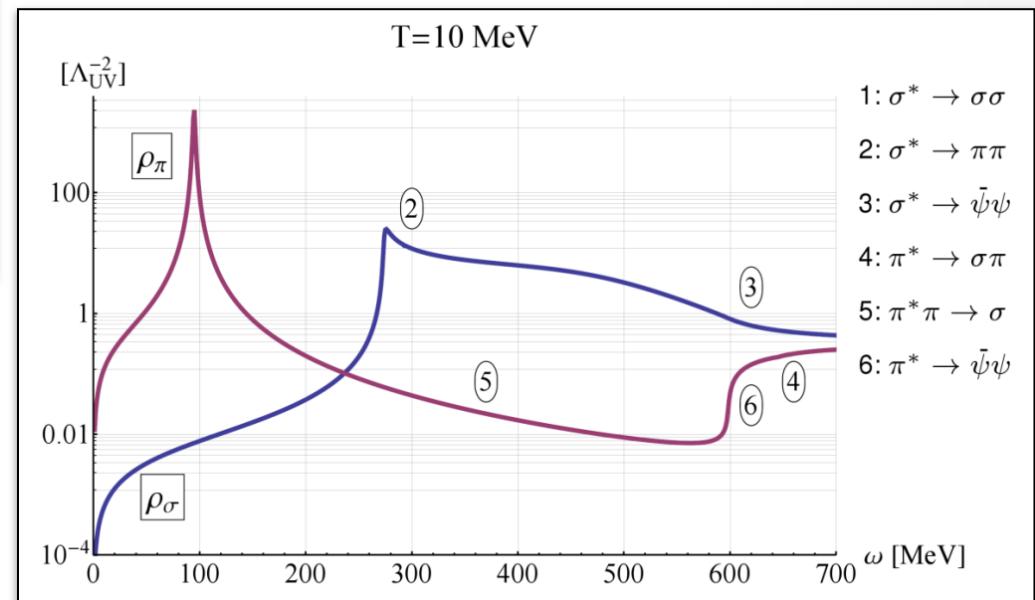
Mesonic Spectral Functions

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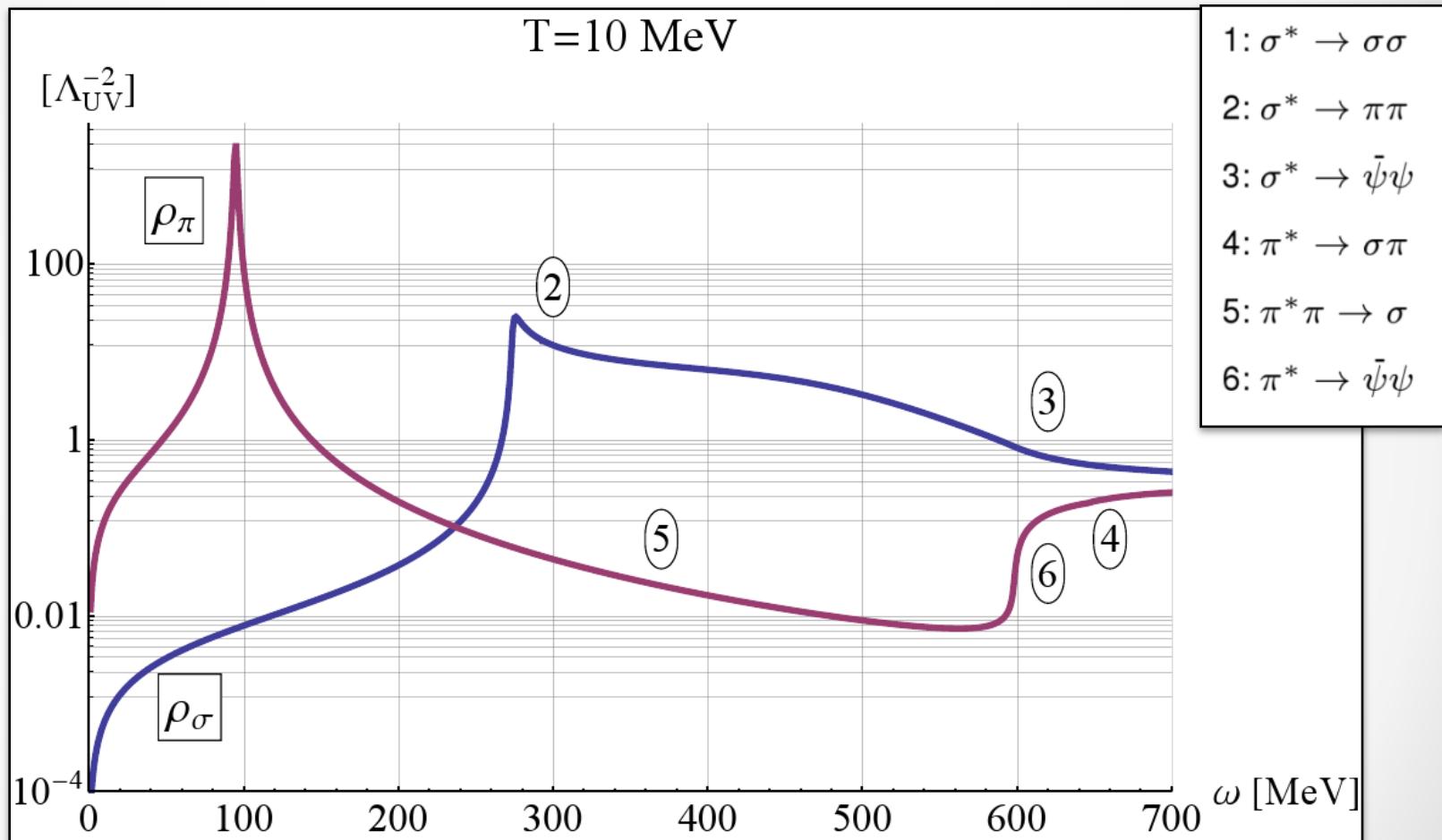
➤ Kamikado, NSt, von Smekal, Wambach
Eur.Phys.J. **C74** (2014) 2806

Quark-meson (Yukawa) model



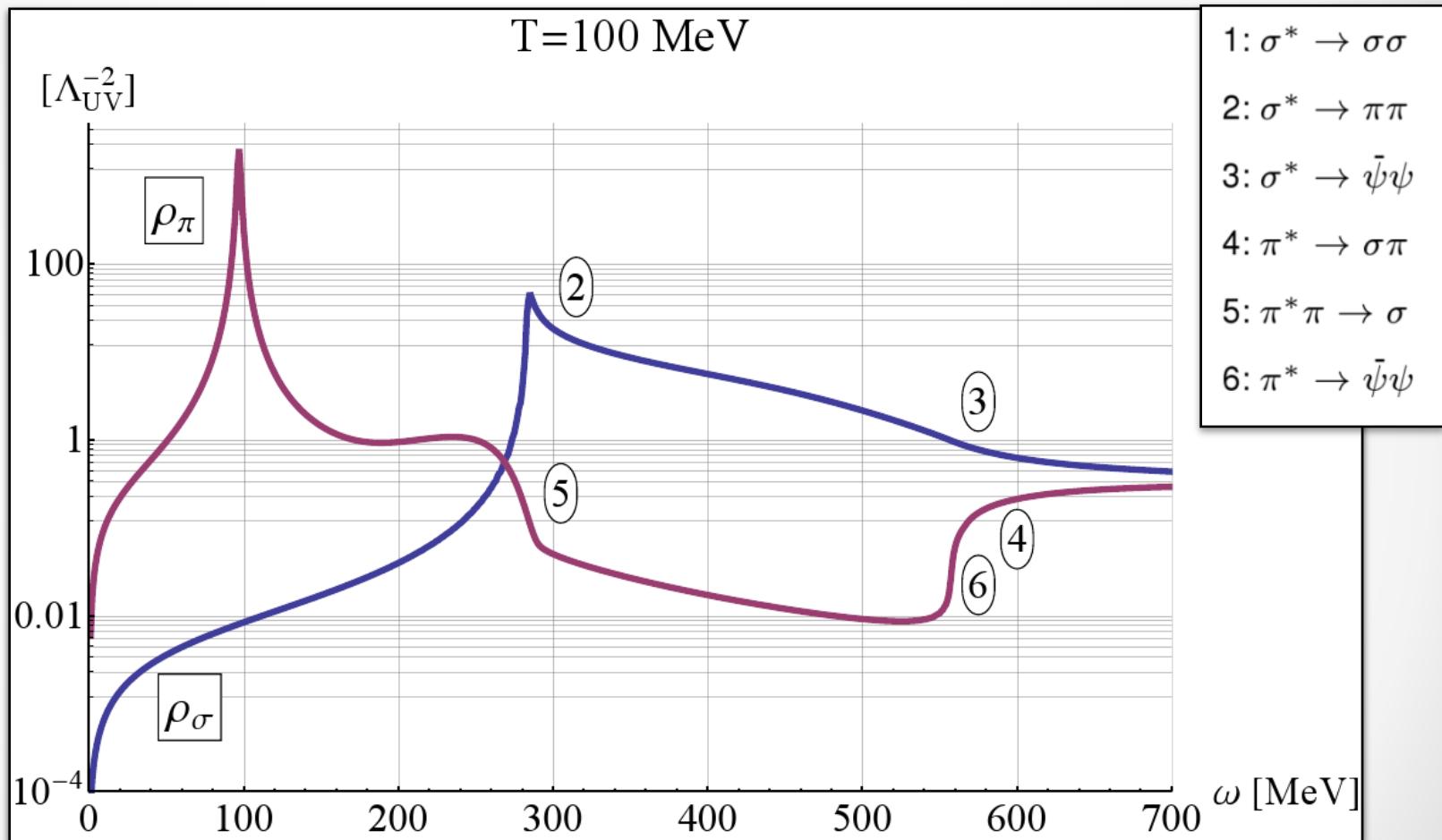
➤ Tripolt, NSt, von Smekal, Wambach
Phys.Rev. **D89** (2014) 034010

QM Model at T>0



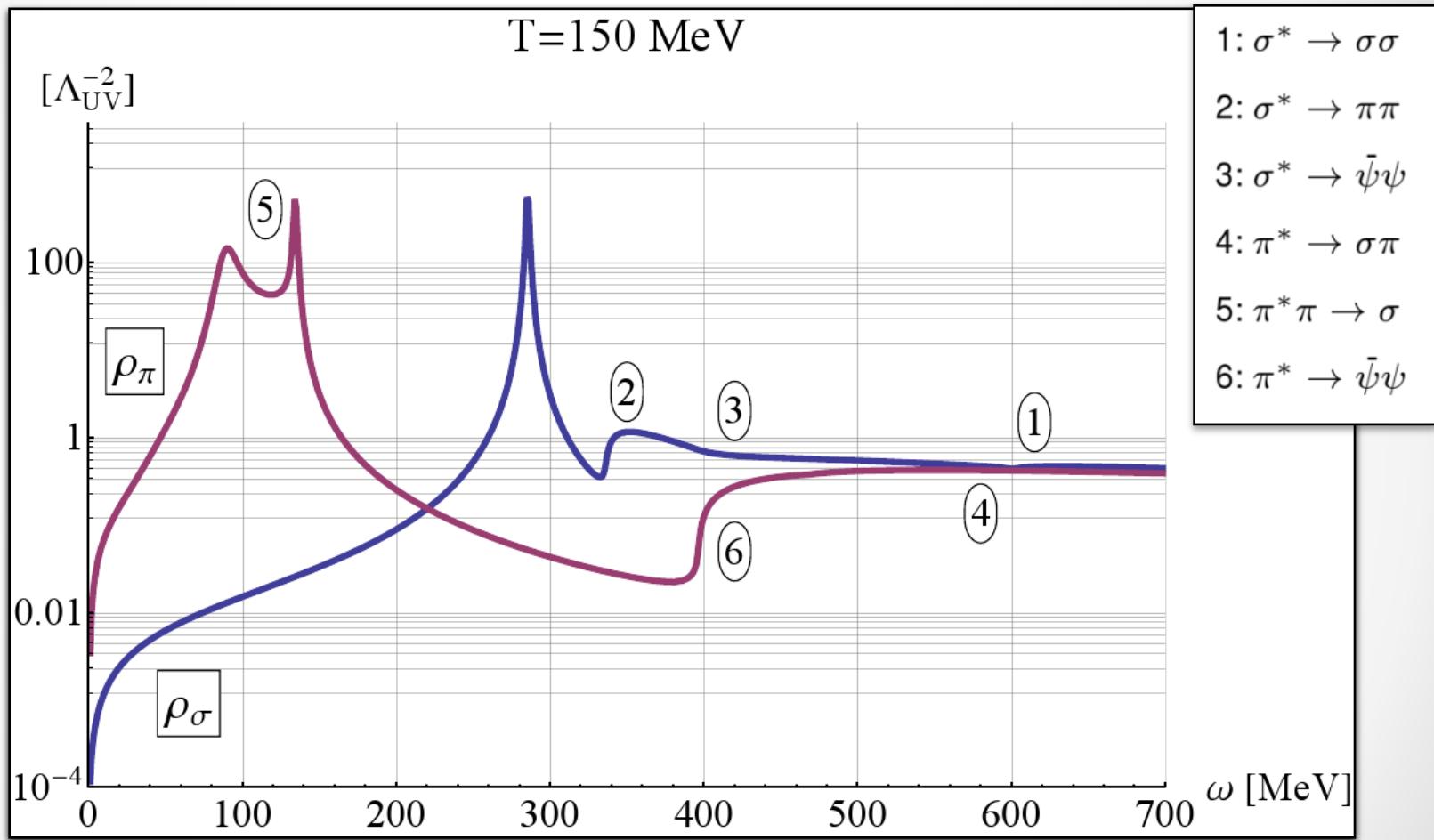
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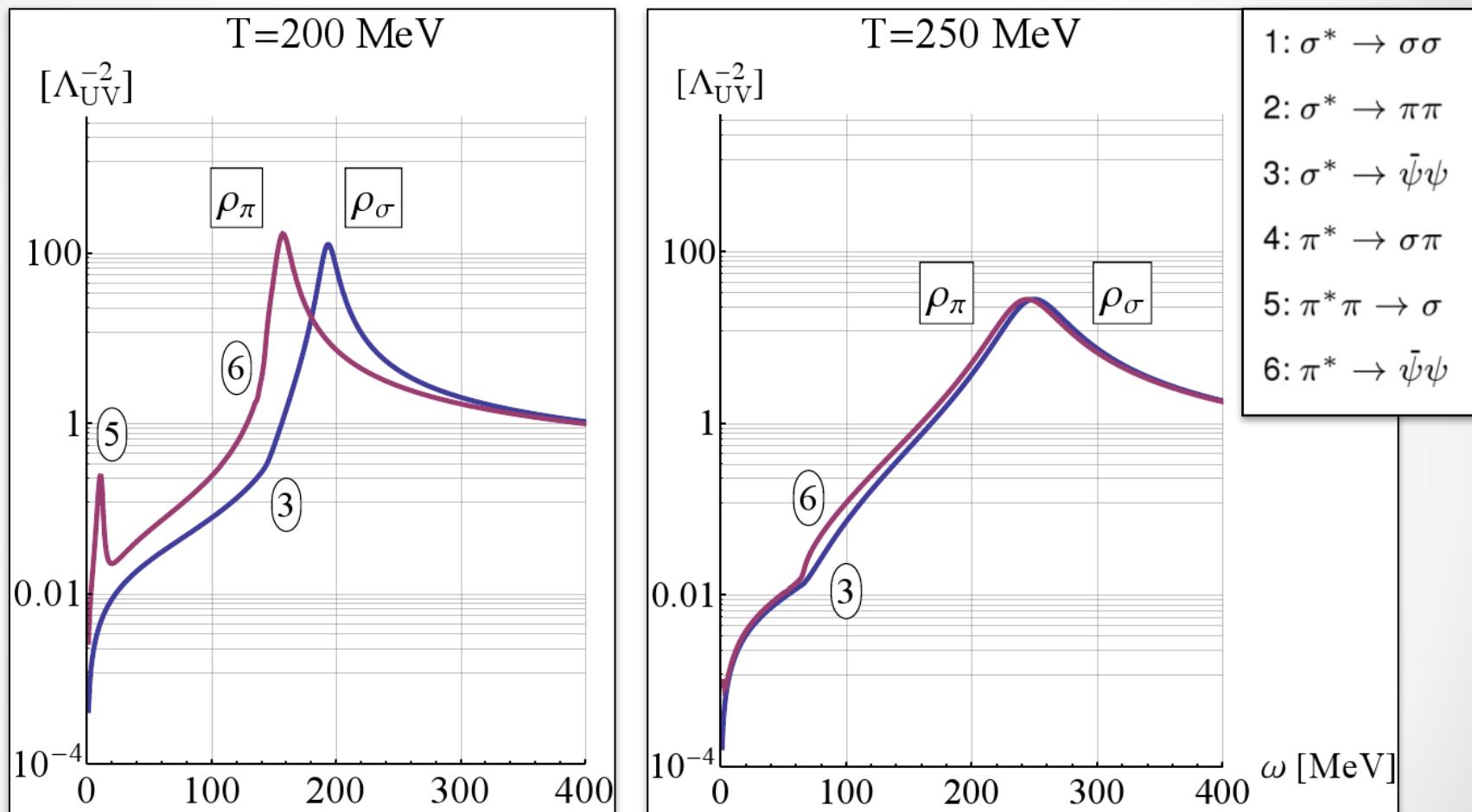
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Outlook



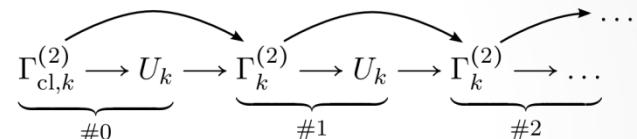
Outlook

Generalization towards a fully numerical procedure

First step: Euclidean momenta (via an iterative procedure)

➤ Helmboldt, Pawłowski, NSt arXiv:1409.8414

step	m_{cur} [MeV]	m_{pol} [MeV]	σ_{\min} [MeV]
0	412.8	412.8	16.8
1	144.8	142 ± 2	83.5
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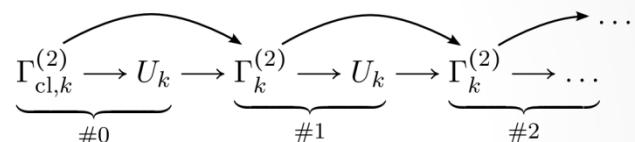
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Second step: Minkowski external momenta

- Pawłowski, NSt in prep.

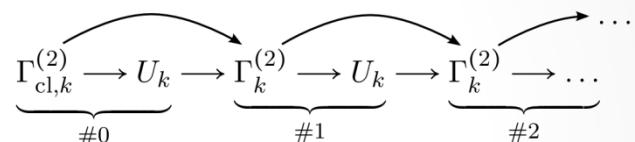
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Shopping list

- ✓ Continuation procedure set-up
- ✓ Tested in simple models
- ❑ Generalization towards a fully numerical procedure
- ❑ Quark & gluon spectral functions
- ❑ Vector meson spectral functions
- ❑ Charmonium spectral functions

Dynamics II

Transport Coefficients

• • •

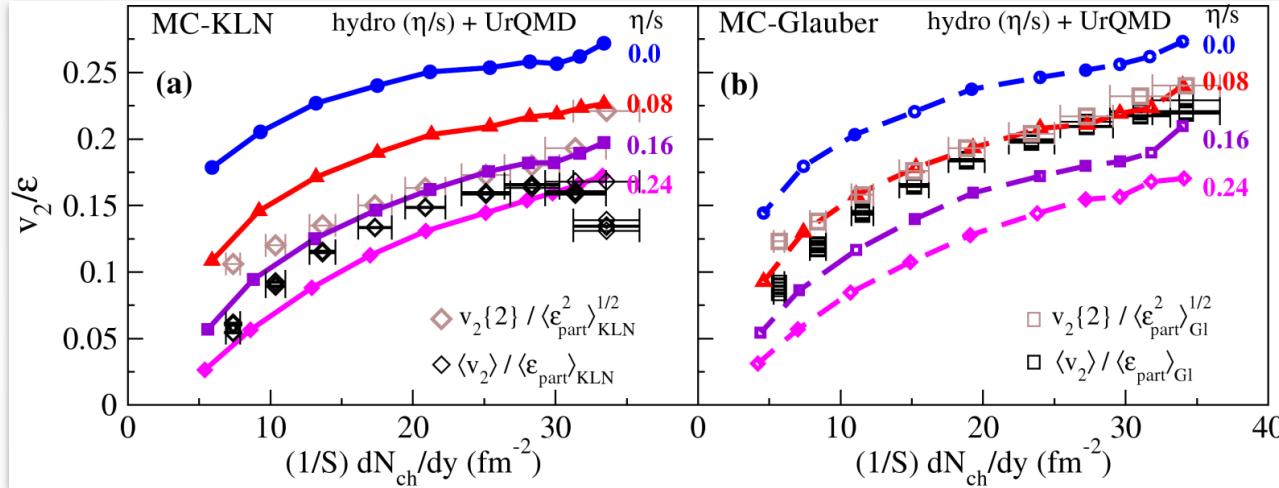
- Christiansen, Haas, Pawłowski , NSt arXiv:1411.7986

Transport Coefficients

- Evolution of the hot plasma well-described by hydrodynamics
- Extract viscosity from ν_2
- Transport coefficients as important microscopic input

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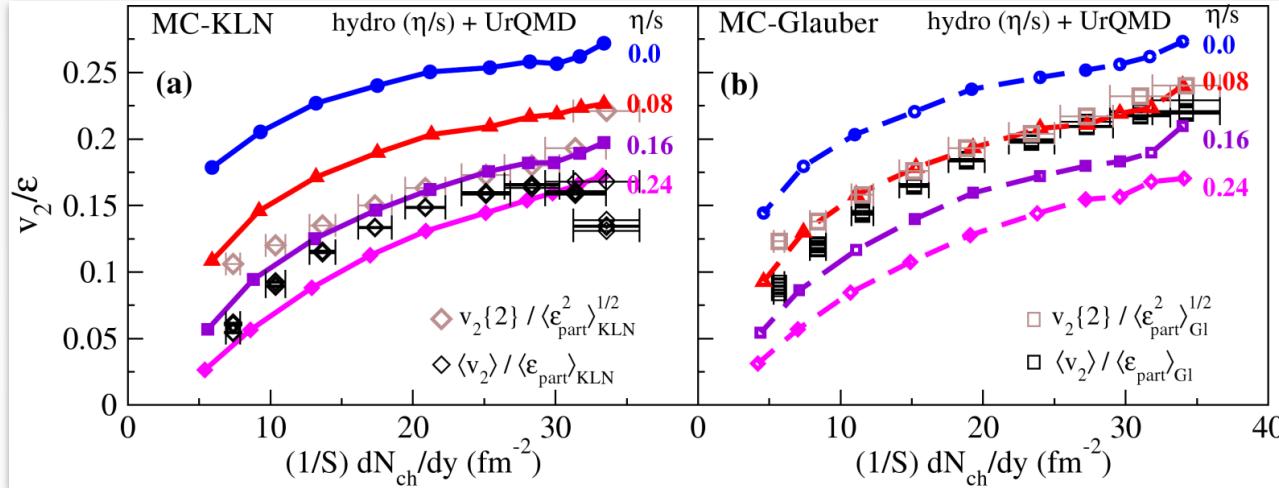
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➤ Bass, Heinz, Hirano, Shen Phys.Rev.Lett. **106** (2011) 192301

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➤ Bass, Heinz, Hirano, Shen Phys.Rev.Lett. **106** (2011) 192301

Kubo formula

$$\eta = \lim_{\omega \rightarrow 0} \frac{1}{20} \frac{\rho_{\pi\pi}(\omega, \vec{0})}{\omega}$$

Require $\rho_{\pi\pi}(\omega, \vec{p}) = \int_x e^{-i\omega x_0 + i\vec{p}\vec{x}} \langle [\pi_{ij}(x), \pi_{ij}(0)] \rangle$

Computing EM Correlators

DSE-like expansion formula

➤ Pawłowski Annals Phys. 322 (2007) 2831-2915

$$\langle \pi_{ij}[\hat{A}] \pi_{ij}[\hat{A}] \rangle = \pi_{ij}[G_{A\phi_k} \frac{\delta}{\delta \phi_k} + A] \pi_{ij}[G_{A\phi_k} \frac{\delta}{\delta \phi_k} + A]$$

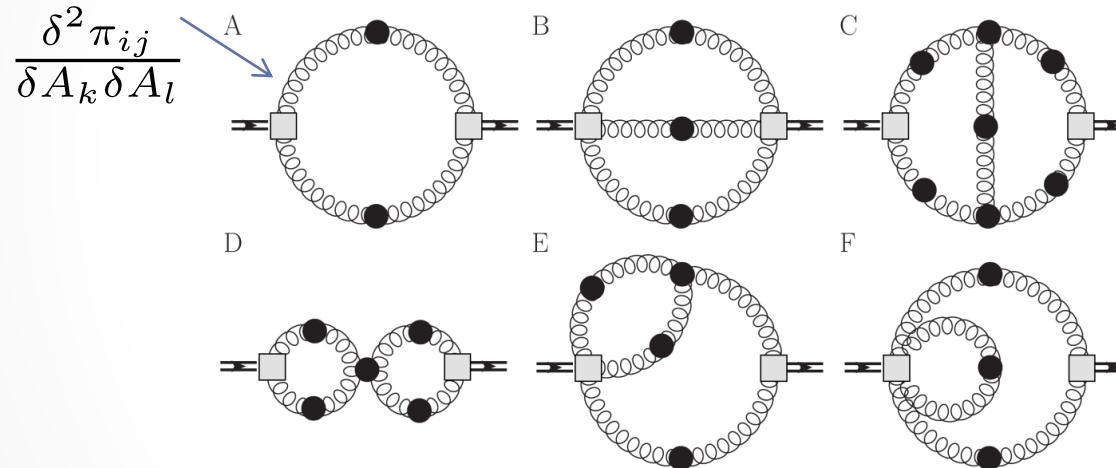
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Finite number of diagrams involving **full** propagators/vertices



All diagrams to
2-loop order

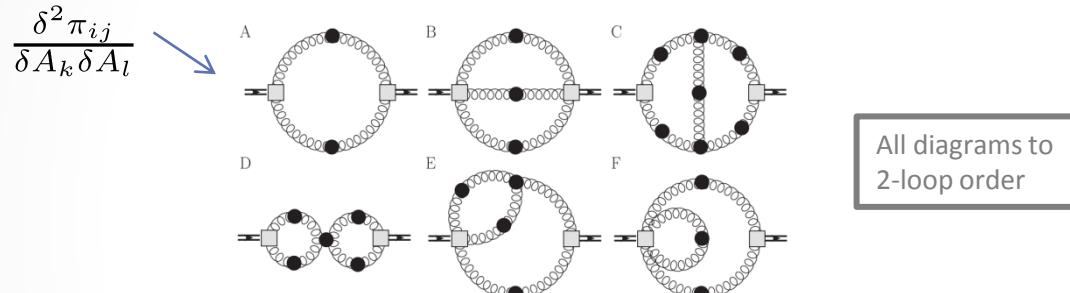
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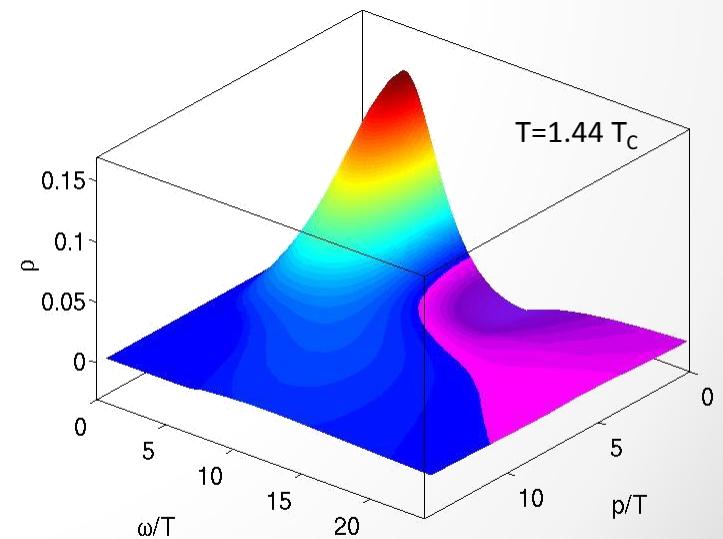
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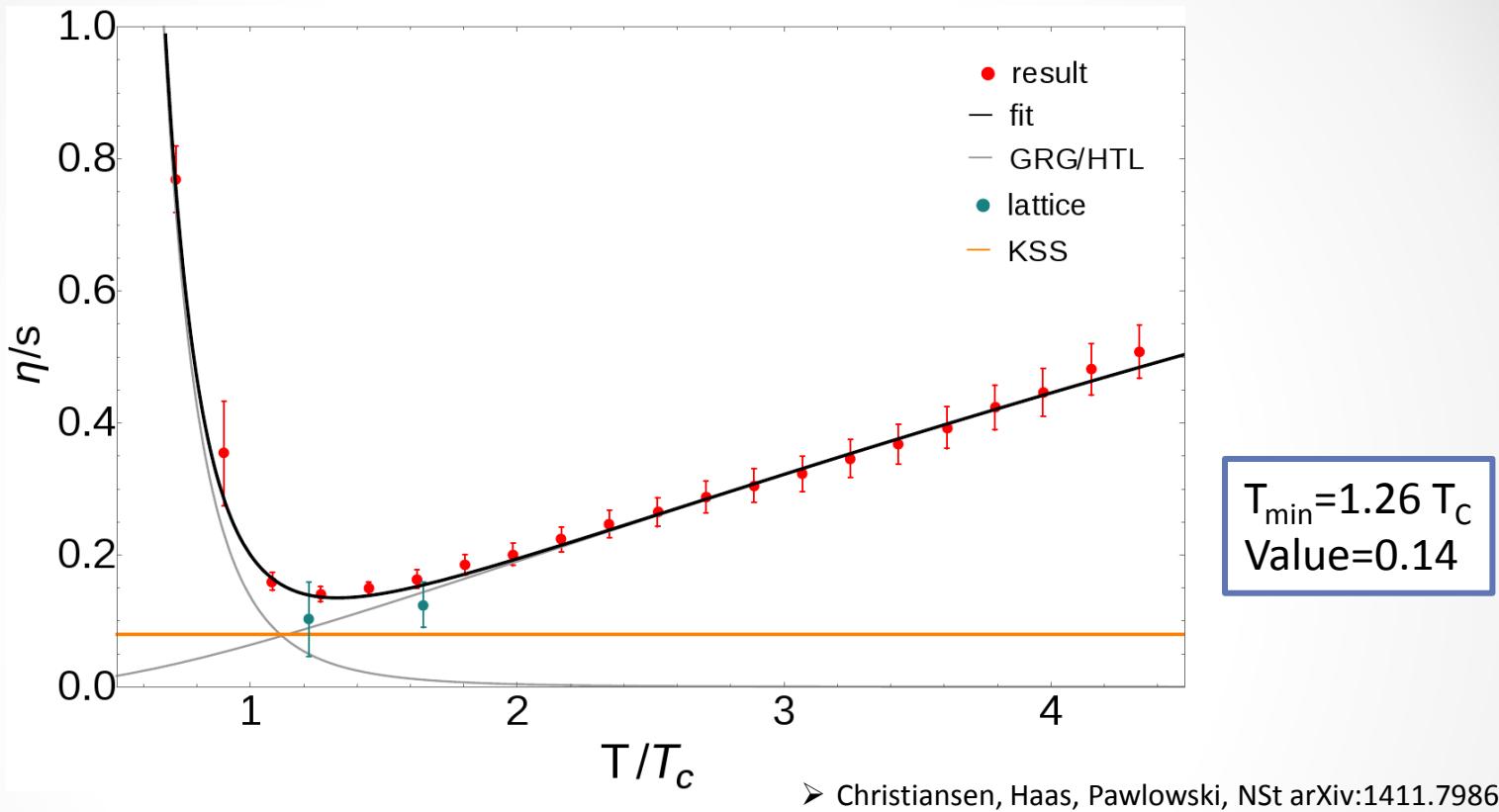


Input: **gluon spectral function** from
Euclidean FRG data using MEM

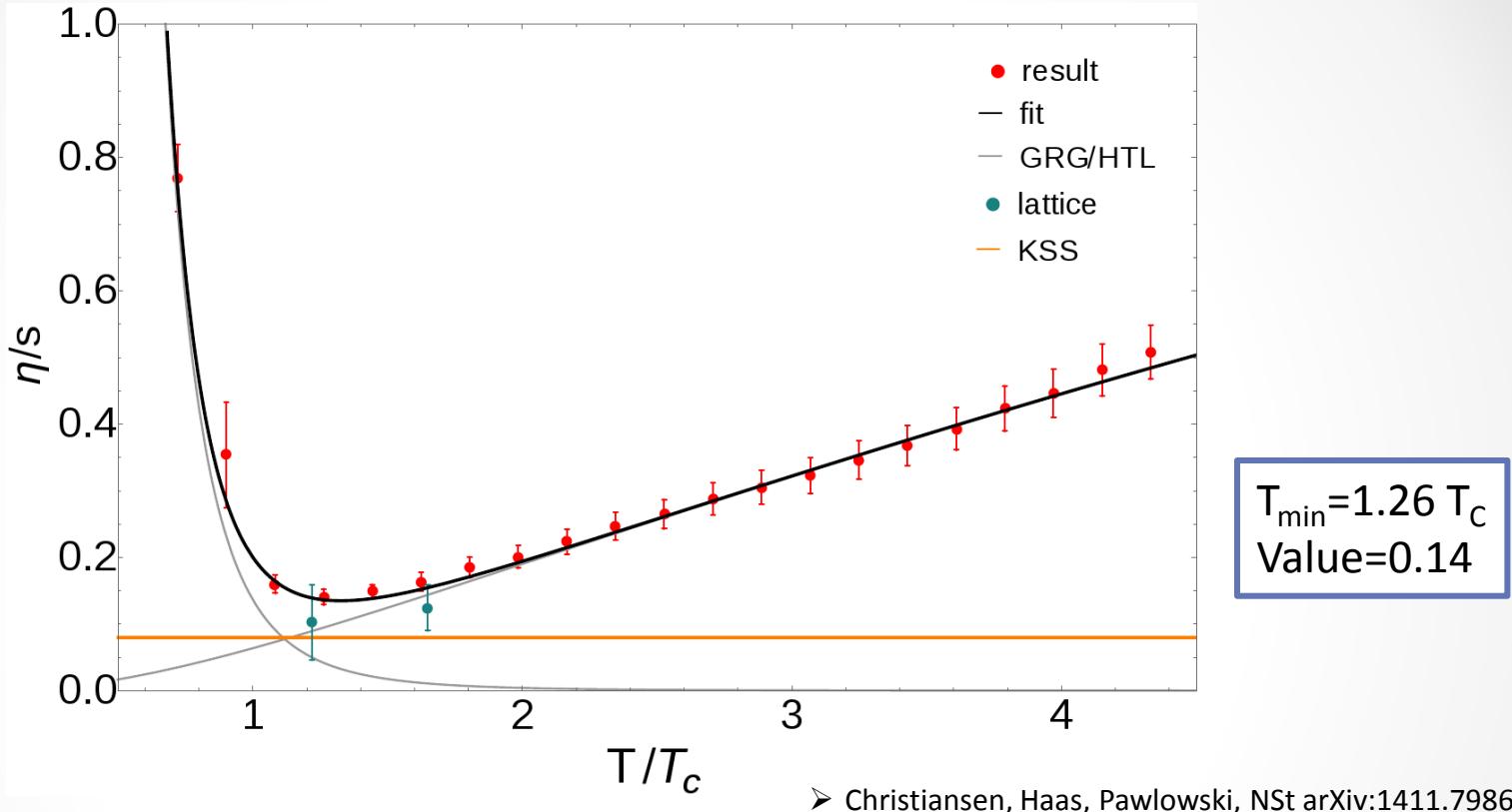
- Haas, Fister, Pawłowski Phys. Rev. D90 091501 (2014)



η/s in Yang-Mills Theory



η/s in Yang-Mills Theory

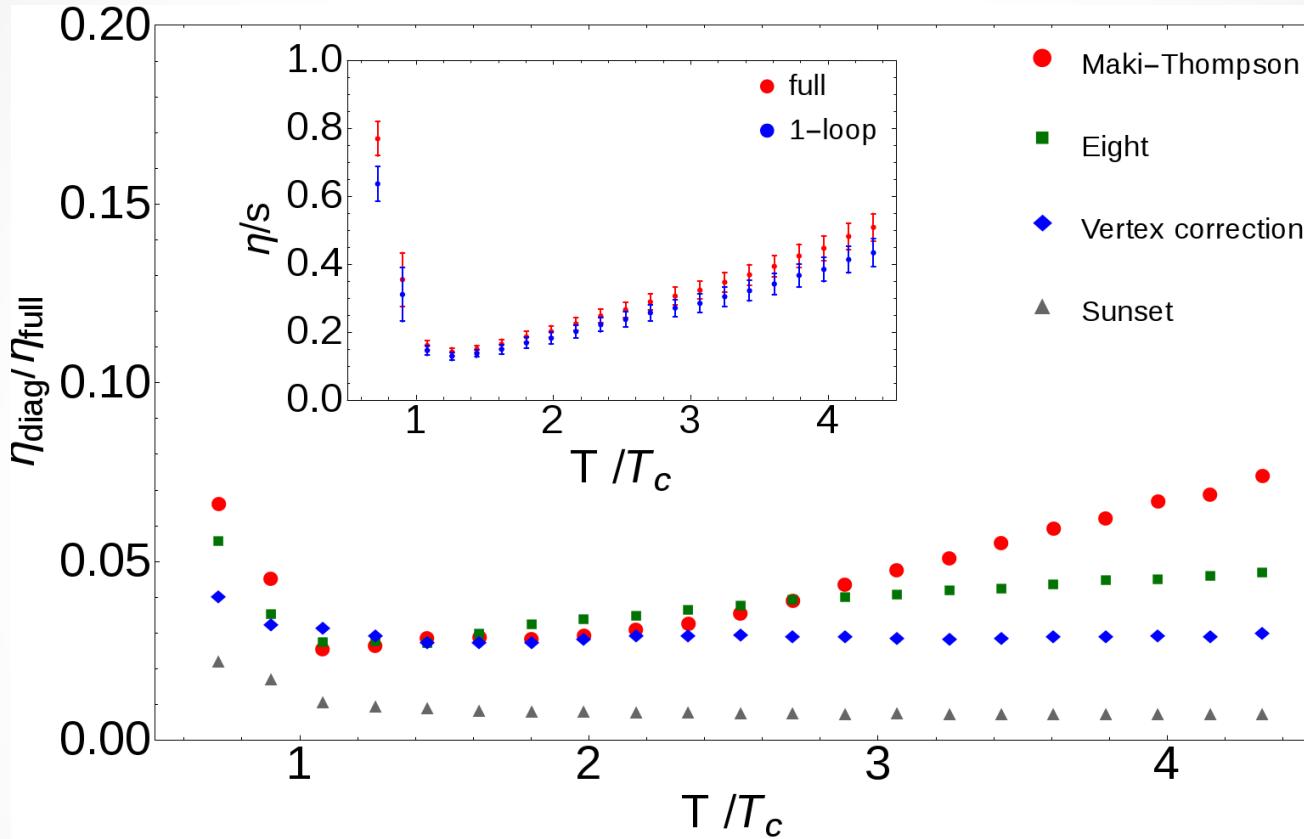


Direct sum:
$$\frac{\eta}{s}(T) = \frac{a}{\alpha_s^\gamma} + \frac{b}{(T/T_c)^\delta}$$

High T: consistent with
HTL-resummed pert. theory
supporting quasiparticle picture

Small T: algebraic decay
glueball resonance gas

More 2-Loop



- Consistent with 1-loop around T_c
- Dominant contribution from Maki-Thompson and Eight at large T

η/s in QCD

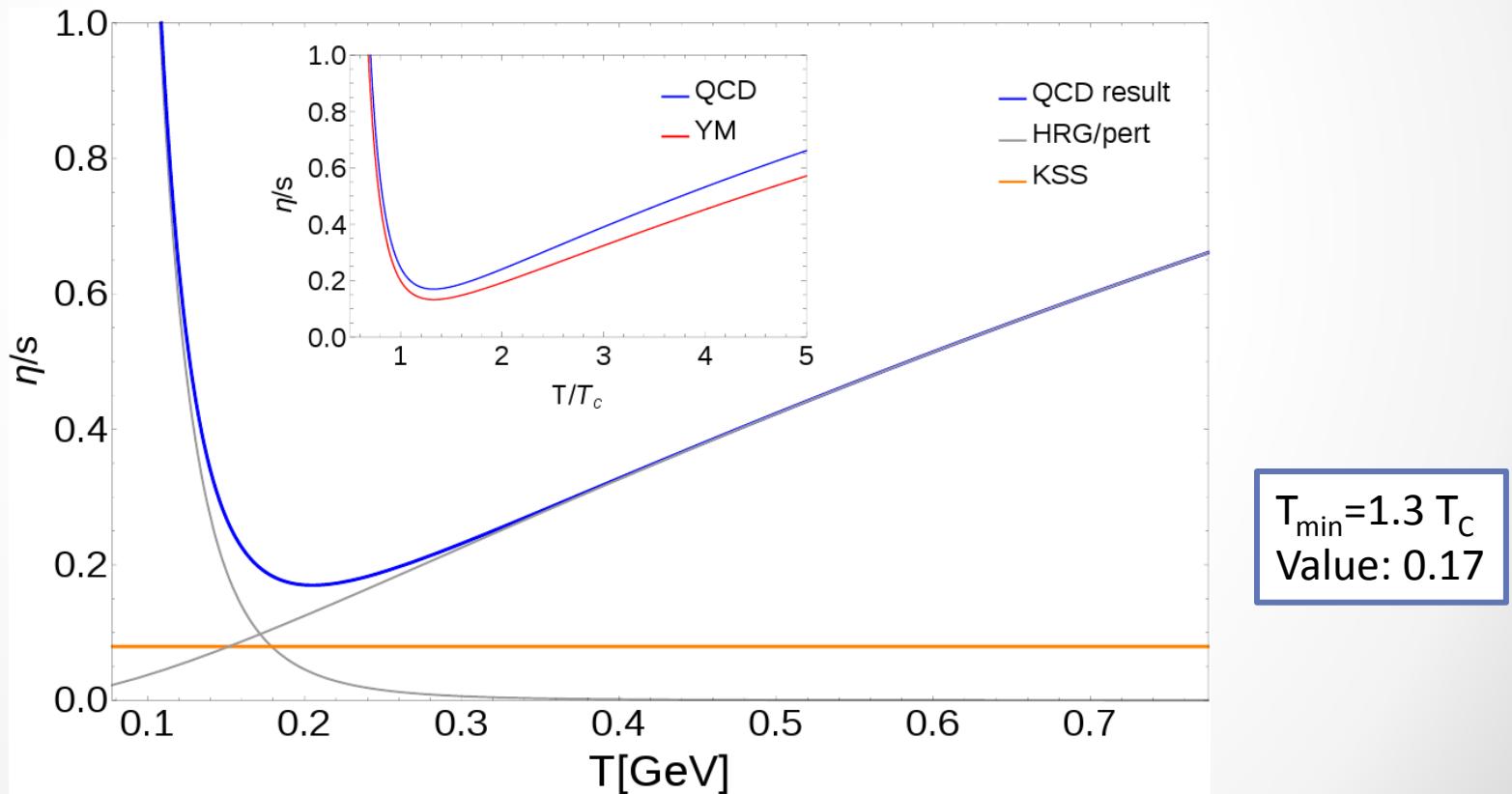
From YM to QCD in three simple steps

1. Replace α_s ; impose equality at T_c $\alpha_s^{N_f=0}|_{T_c} = \alpha_s^{N_f=3}|_{T_c}$
2. Genuine quark contributions to η and s
3. Replace GRG by HRG ➤ Demir, Bass PRL **102** (2009) 172302

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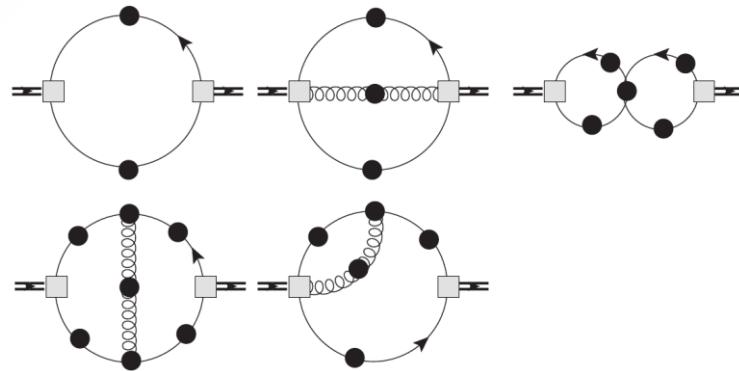
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Outlook

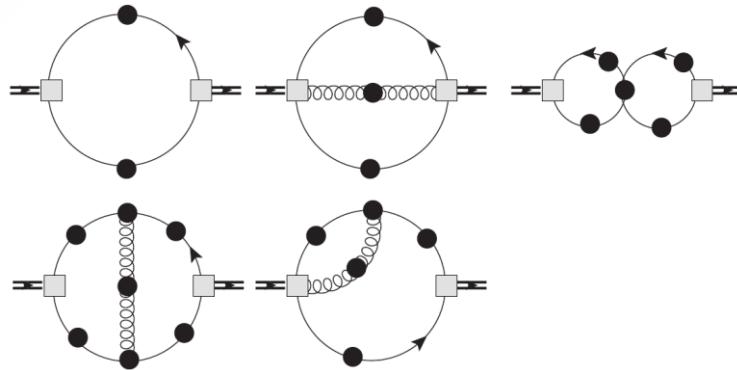
Quark contributions



Require quark and gluon spectral functions in QCD

Outlook

Quark contributions



Require quark and gluon spectral functions in QCD

Shopping List

- ✓ Formalism set-up
- ✓ Quantitative results for η/s in YM
- ❑ Bulk viscosity
- ❑ Relaxation times
- ❑ Application to non-relativistic systems e.g. ultracold atoms

Summary

- **QCD phase structure**

towards a quantitative continuum approach to QCD

- ✓ Quantitative grip on fluctuation physics in the vacuum
- ❑ finite temperature and density

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new approach to analytical continuation problem

- ✓ tested in simple models ($O(N)$, QM model)
- ❑ quark & gluon spectral functions, vector mesons, charmonium

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- Thank you for your attention!

-

Backup

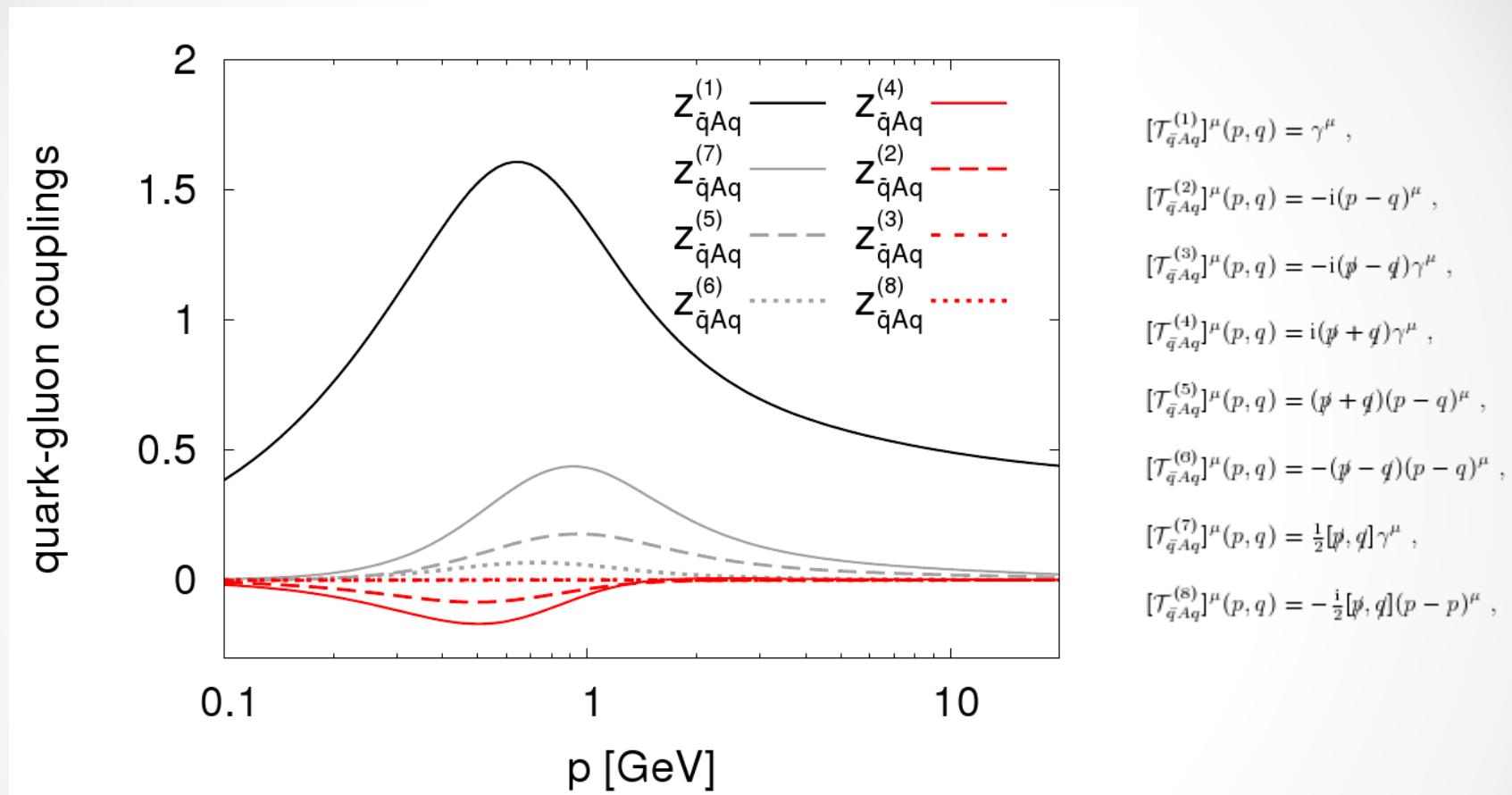
• • •

More Matter system?

• • •

- Mitter, Pawłowski, NSt arXiv:1411.7978

Quark-Gluon vertex



- Take into account all 8 tensor structures of the trans. projected vertex
- 5,7 most important (non-classical) tensor structures in the symmetric phase
- **but keep in mind gauge-invariance**

STI-consistent expansion

Setting up a sensible truncation scheme:

- **Use an expansion in terms of gauge-invariant operators**
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- e.g. for the dominant (non-classical) structure in the chirally symmetric regime: gauge invariant operator

$$i\sqrt{4\pi\alpha_s}\bar{q}\gamma_5\gamma_\mu\epsilon_{\mu\nu\rho\sigma}\{F_{\nu\rho}, D_\sigma\}q$$

gives rise to contribution proportional to

$$\frac{1}{2}\mathcal{T}_{\bar{q}Aq}^{(5)} + \mathcal{T}_{\bar{q}Aq}^{(7)}$$

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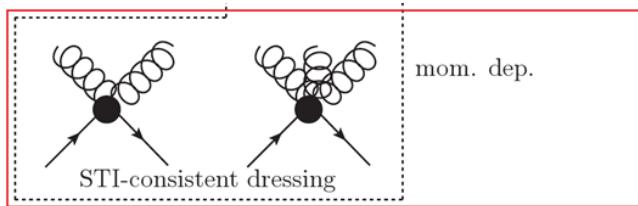
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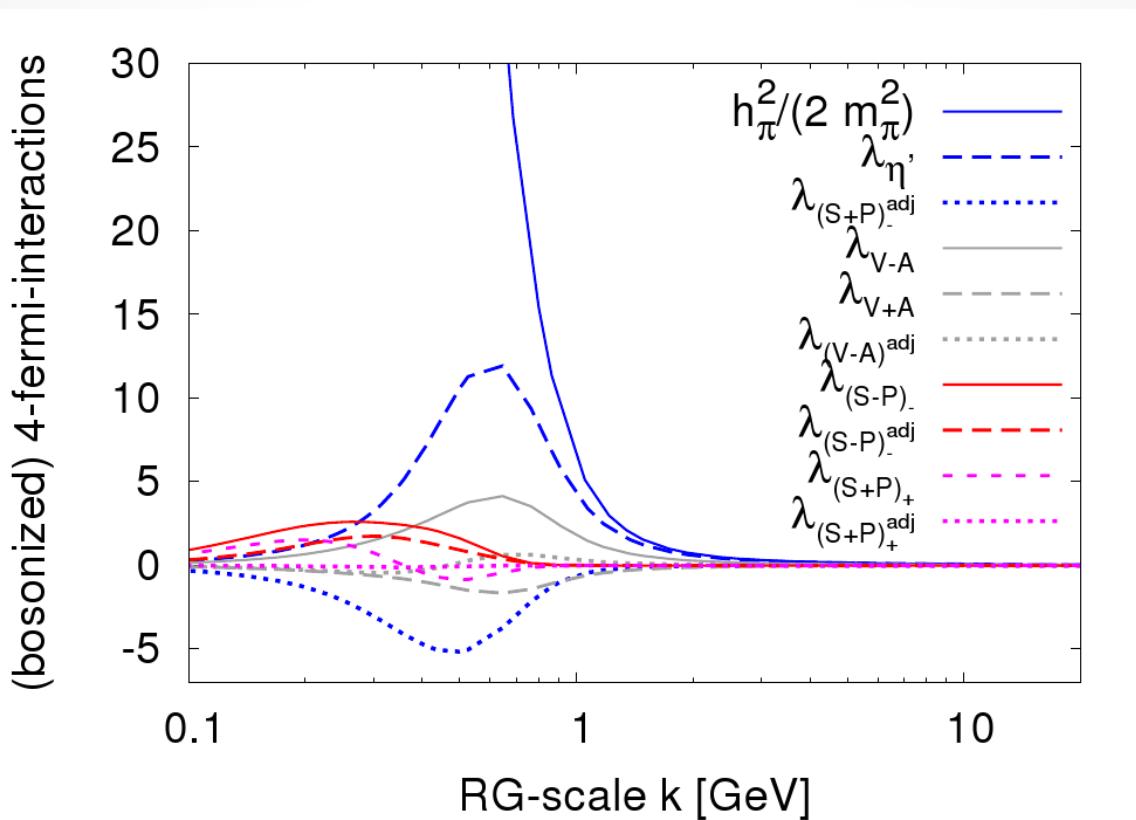
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- Associated non-classical vertices are quantitatively important

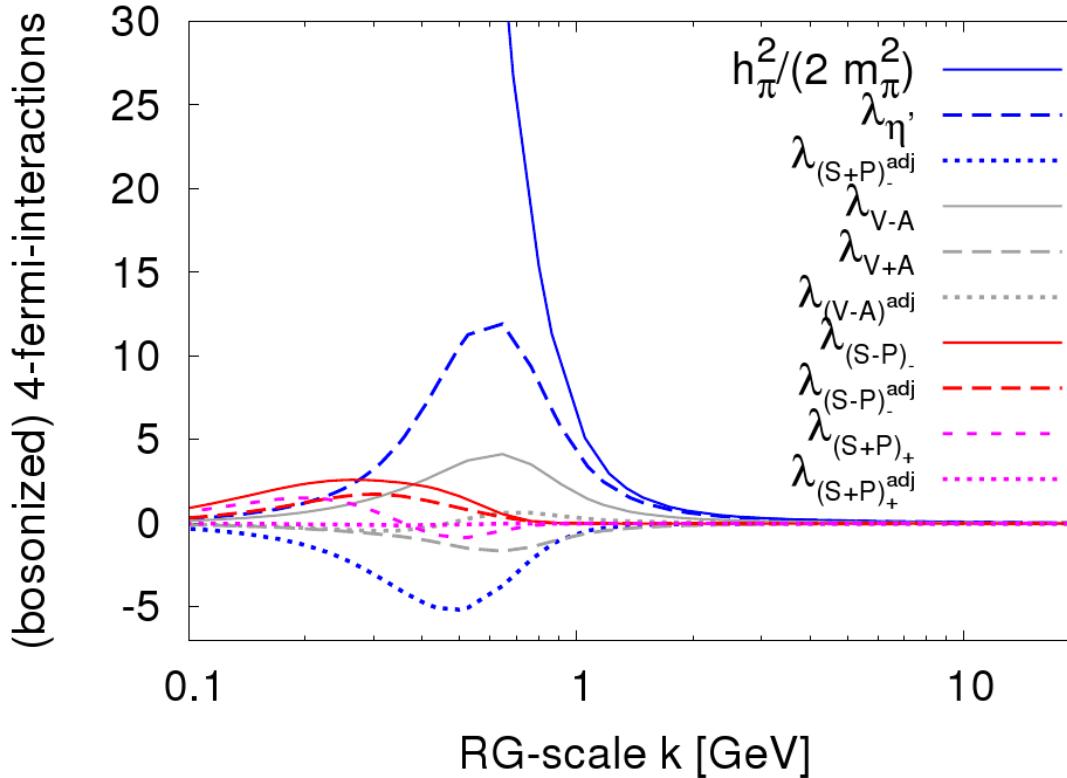


4-Fermi Interactions



(a) Renormalisation group scale dependence of dimensionless four-fermi interactions, see App. B 2 c, and bosonised σ - π channel. Grey: respects chiral symmetry, blue: breaks $U(1)_A$, red: breaks $SU(2)_A$, magenta: breaks $U(2)_A$.

4-Fermi Interactions



(a) Renormalisation group scale dependence of dimensionless four-fermi interactions, see App. B 2 c, and bosonised σ - π channel. Grey: respects chiral symmetry, blue: breaks $U(1)_A$, red: breaks $SU(2)_A$, magenta: breaks $U(2)_A$.

- **Bosonizing the σ - π channel only is sufficient to remove divergence**
- In the vacuum: other channels not quantitatively relevant

Euclidean Iteration

• • •

- Helmboldt, Pawłowski, NSt arXiv:1409.8414

Why momentum dependence?

Quantitative precision



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- QCD perspective on low-energy effective models:
 - ✓ **UV parameters fixed** by QCD flows
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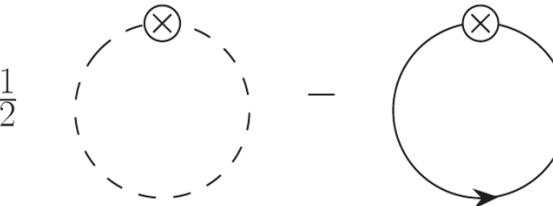
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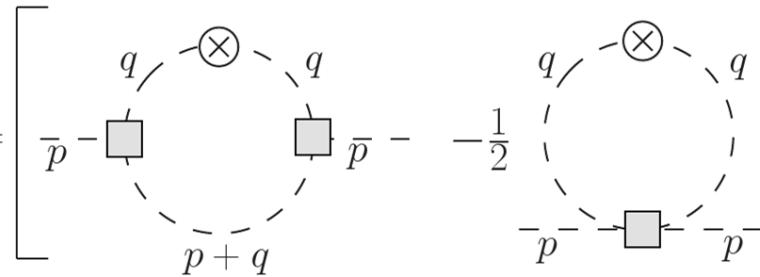
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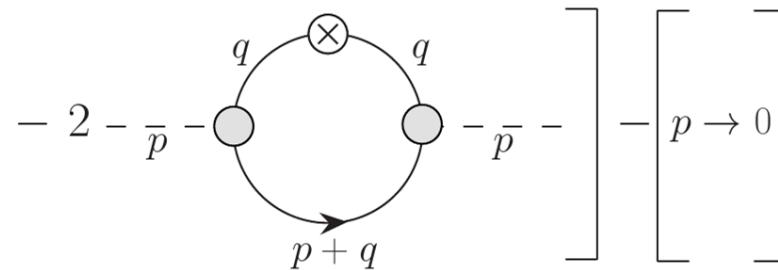
Euclidean Iteration I

Momentum dependence of 2-point functions in an iterative procedure

Example: mesonic propagators in a quark meson model

$$\partial_t U_k = \frac{1}{2} \left(\text{Diagram A} - \text{Diagram B} \right)$$


$$\partial_t \Delta \Gamma_k^{(2)}(p^2) = \begin{bmatrix} q & \text{Diagram C} & q \\ \bar{p} - \square & & \square \bar{p} \\ & \text{Diagram D} & \\ p + q & & p - \end{bmatrix} - \frac{1}{2} \begin{bmatrix} q & \text{Diagram E} & q \\ \bar{p} - \square & & \square \bar{p} \\ & \text{Diagram F} & \\ p - & & p - \end{bmatrix}$$


$$- 2 - \bar{p} - \left[\text{Diagram G} - \bar{p} \right] - \left[\text{Diagram H} - \bar{p} \right] - \left[p \rightarrow 0 \right]$$


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$$\partial_t \Delta \Gamma_k^{(2)}(p^2) = \left[\begin{array}{ccc} q & \otimes & q \\ \bar{p} - \square & & \square \bar{p} \\ p+q & & p-q \end{array} \right] - \frac{1}{2} \left[\begin{array}{c} q & \otimes & q \\ p & & p \\ -p & & p \end{array} \right]$$

momentum-independent vertices from eff. potential

$$- 2 - \bar{p} - \left[\begin{array}{c} q \\ \otimes \\ q \\ \bar{p} \\ p+q \end{array} \right] - \left[\begin{array}{c} p \rightarrow 0 \end{array} \right]$$

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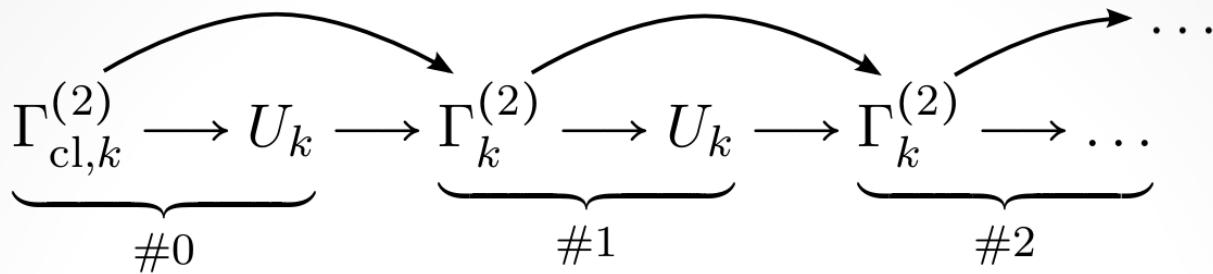
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momentum-independent vertices from eff. potential

Iteration procedure

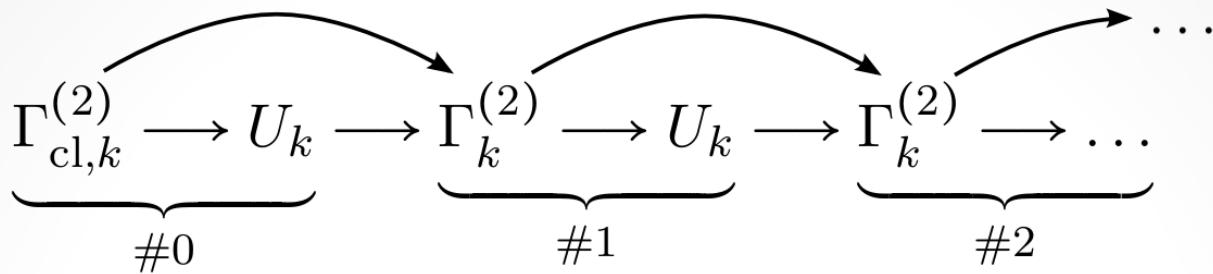
$$\underbrace{\Gamma_{\text{cl},k}^{(2)} \rightarrow U_k}_{\#0} \rightarrow \underbrace{\Gamma_k^{(2)} \rightarrow U_k}_{\#1} \rightarrow \underbrace{\Gamma_k^{(2)} \rightarrow \dots}_{\#2} \dots$$

Euclidean Iteration II



- Numerically inexpensive upgrade for existing Euclidean calculations
- Here: Quark-meson model at finite T; fixed ren. Yukawa coupling
- 4d exponential regulator function

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- **Numerically inexpensive upgrade** for existing Euclidean calculations
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- **4d exponential** regulator function
- Convergence properties:

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Pole mass: $\bar{\Gamma}^{(2)}(\mathrm{i}m_{\text{pol}}, 0) = 0$

Temporal screening: $T \sum_{p_0} \Gamma^{(2)}(p_0, 0)^{-1} e^{\mathrm{i}p_0 t} \sim e^{-m_{\text{pol}}|t|}$

Mass Definitions

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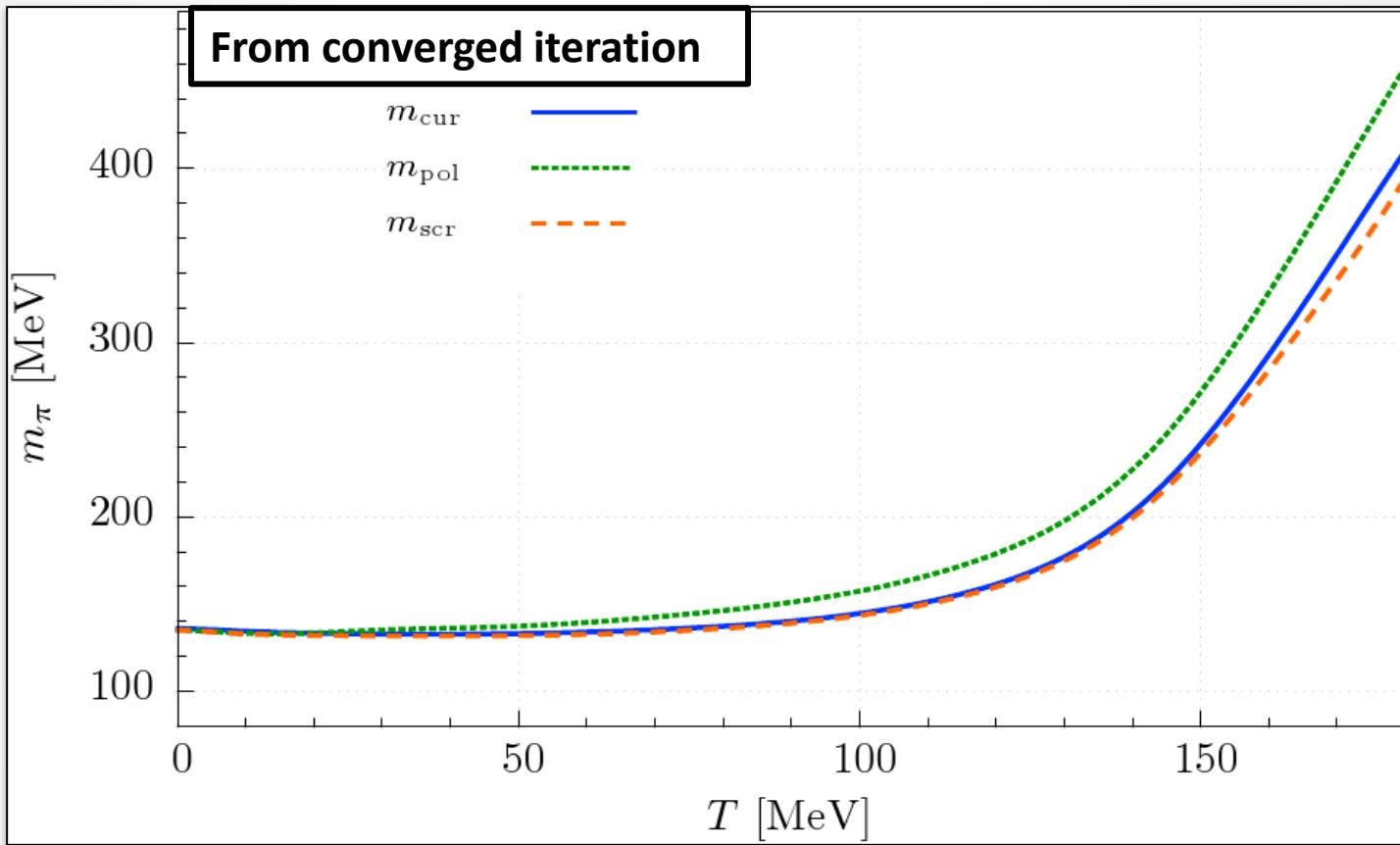
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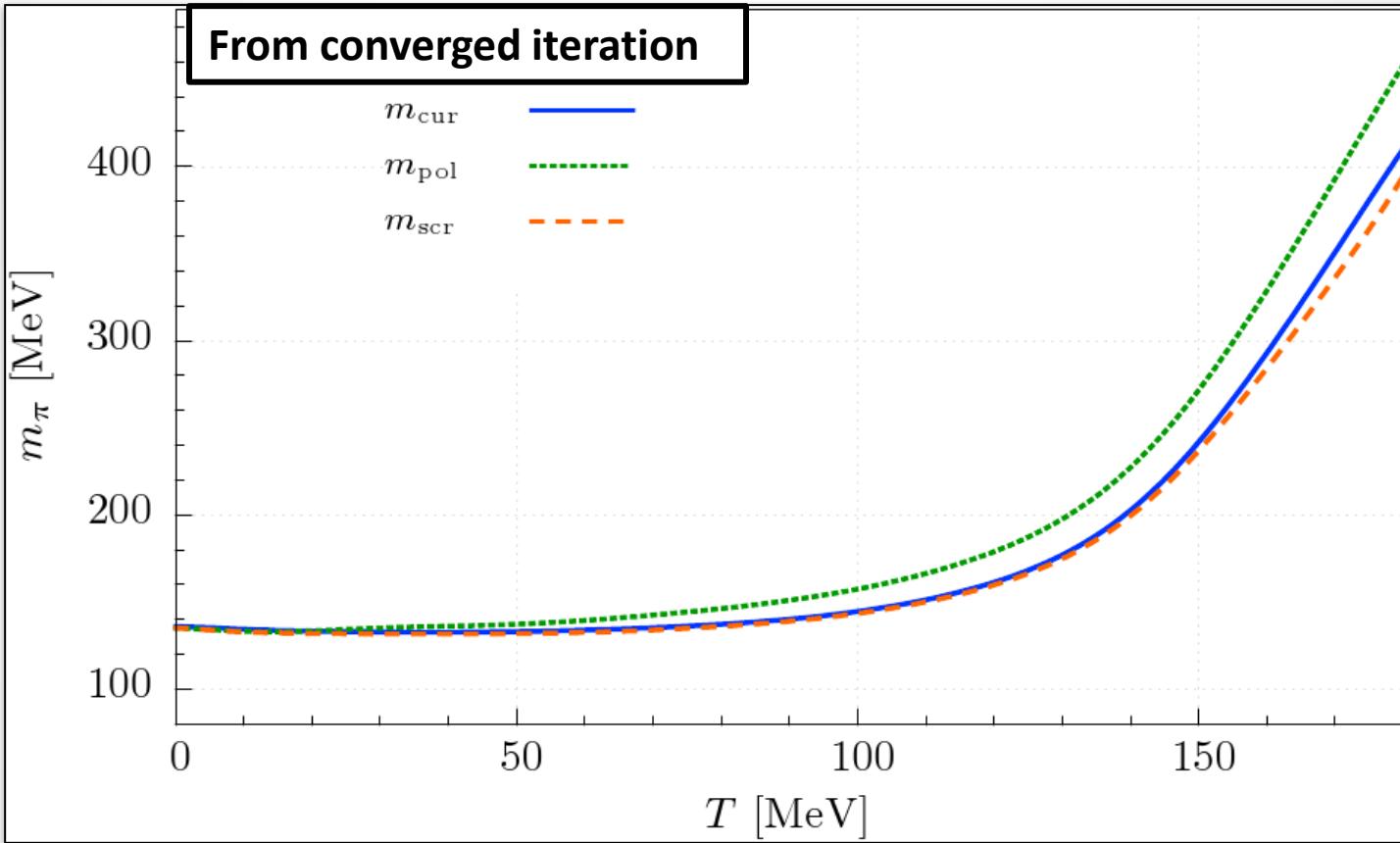
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Onset mass: Silver Blaze property links mass to critical chemical potential; coincides with pole mass

Physics Results



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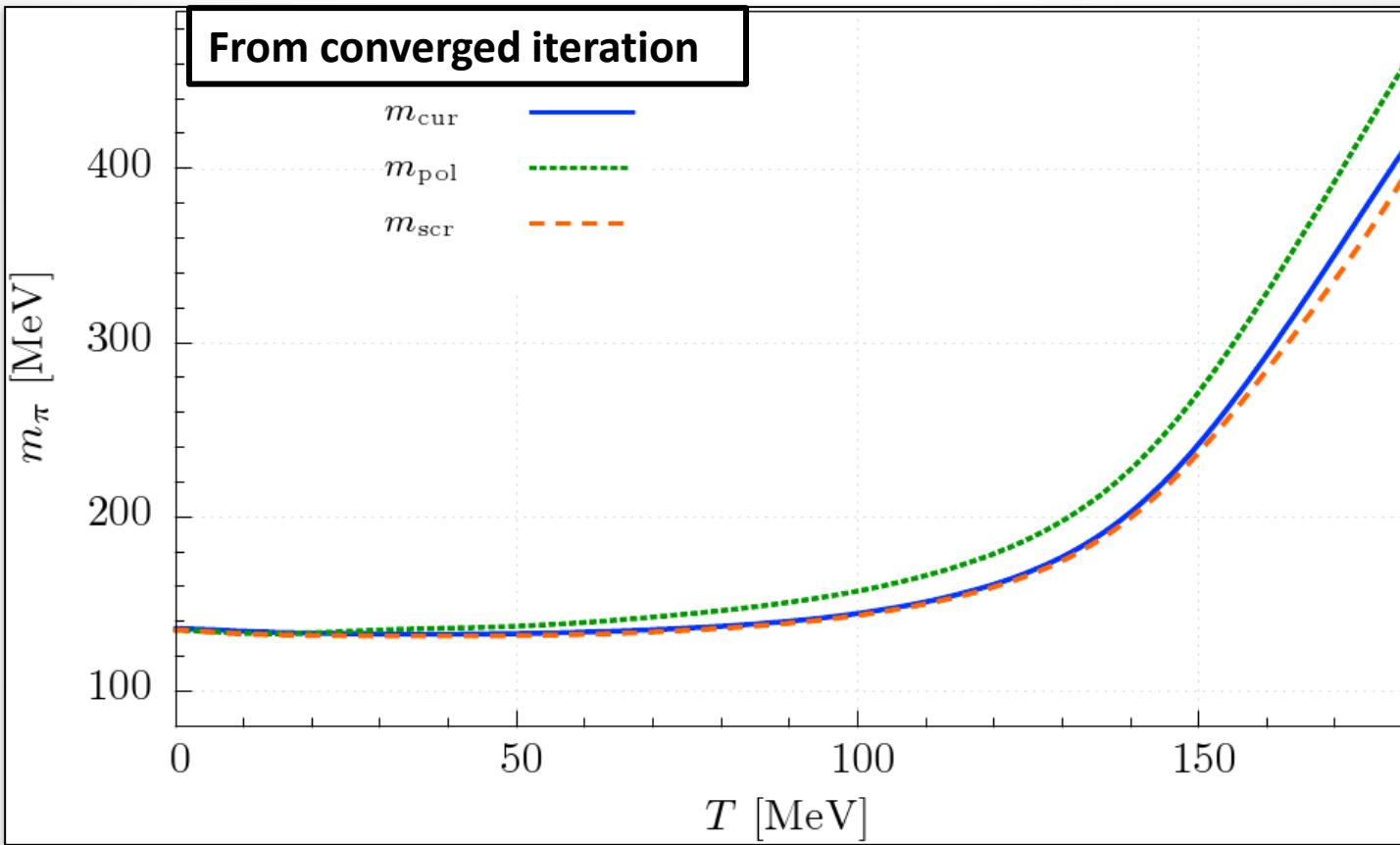


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$m_{\text{pol}} = m_{\text{scr}}$ by O(4) invariance

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LPA: Mismatches of Fluctuation Scales

More than an academic exercise...

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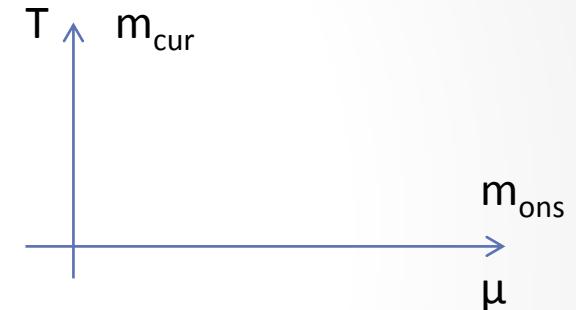
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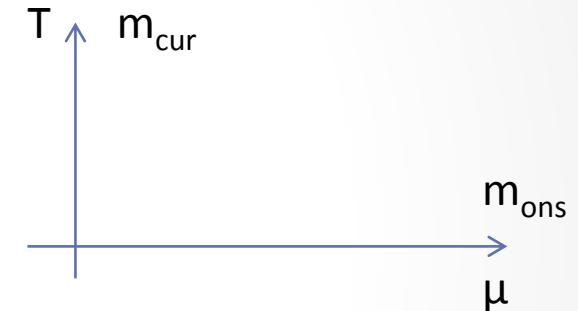
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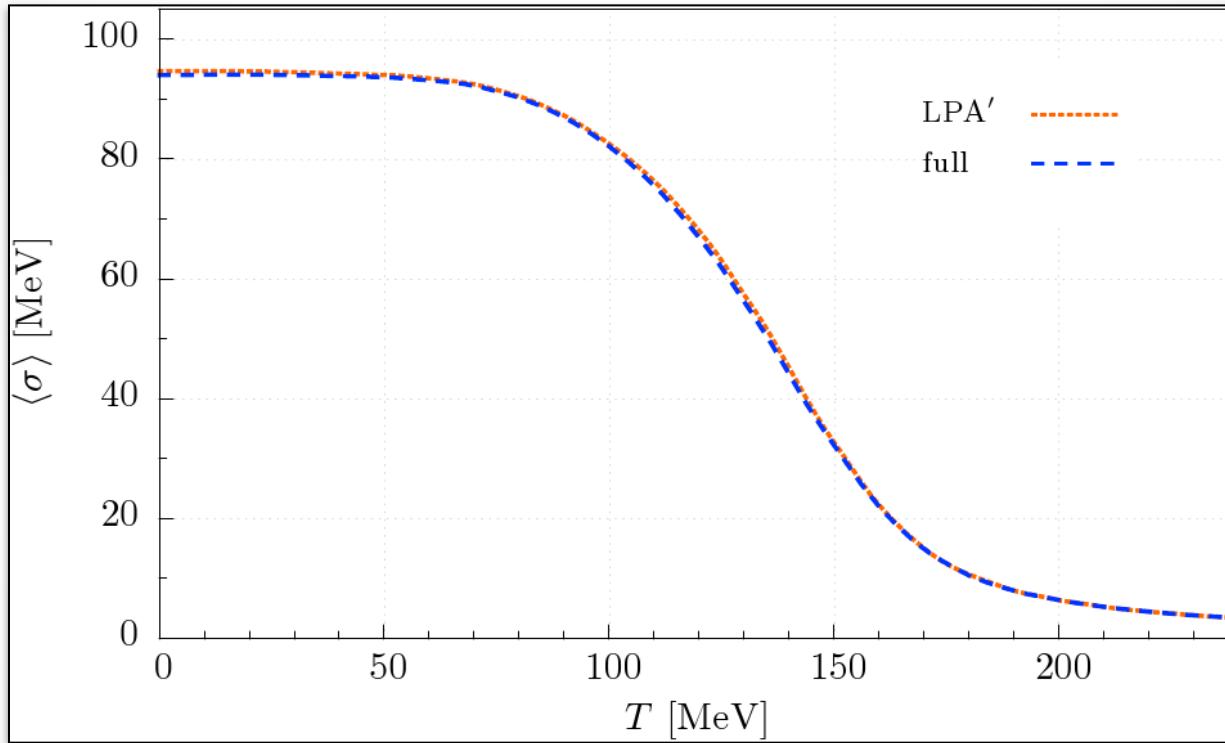
Rough estimate:

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- **mismatch of fluctuation scales**
=> **large systematic errors at finite μ (curvature, CEP)**
- resolved by including momentum dependence

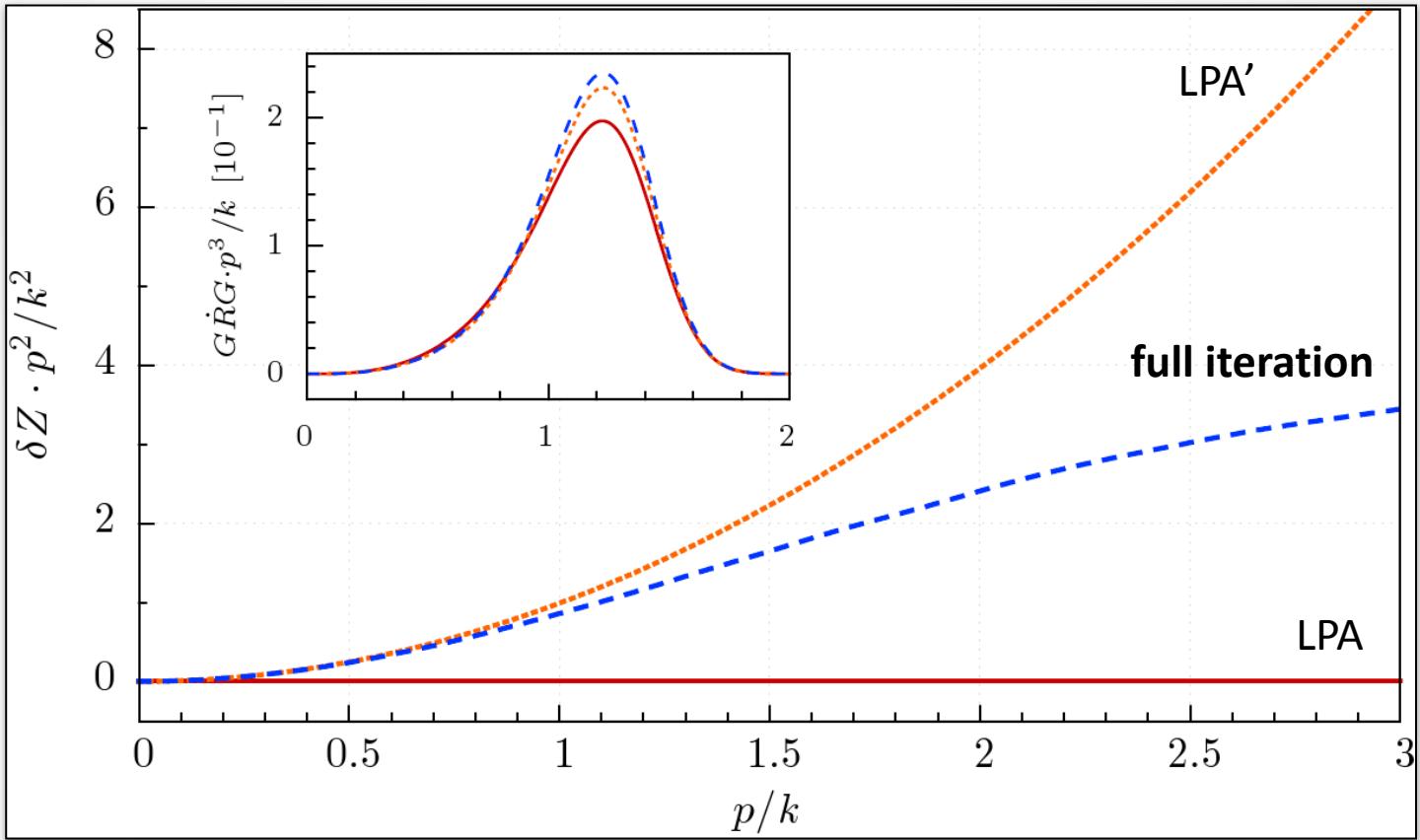
Comparison: Fixed UV

QCD perspective



- LPA with these initial conditions => no χ SB
- Full calculation and LPA' in quantitative agreement

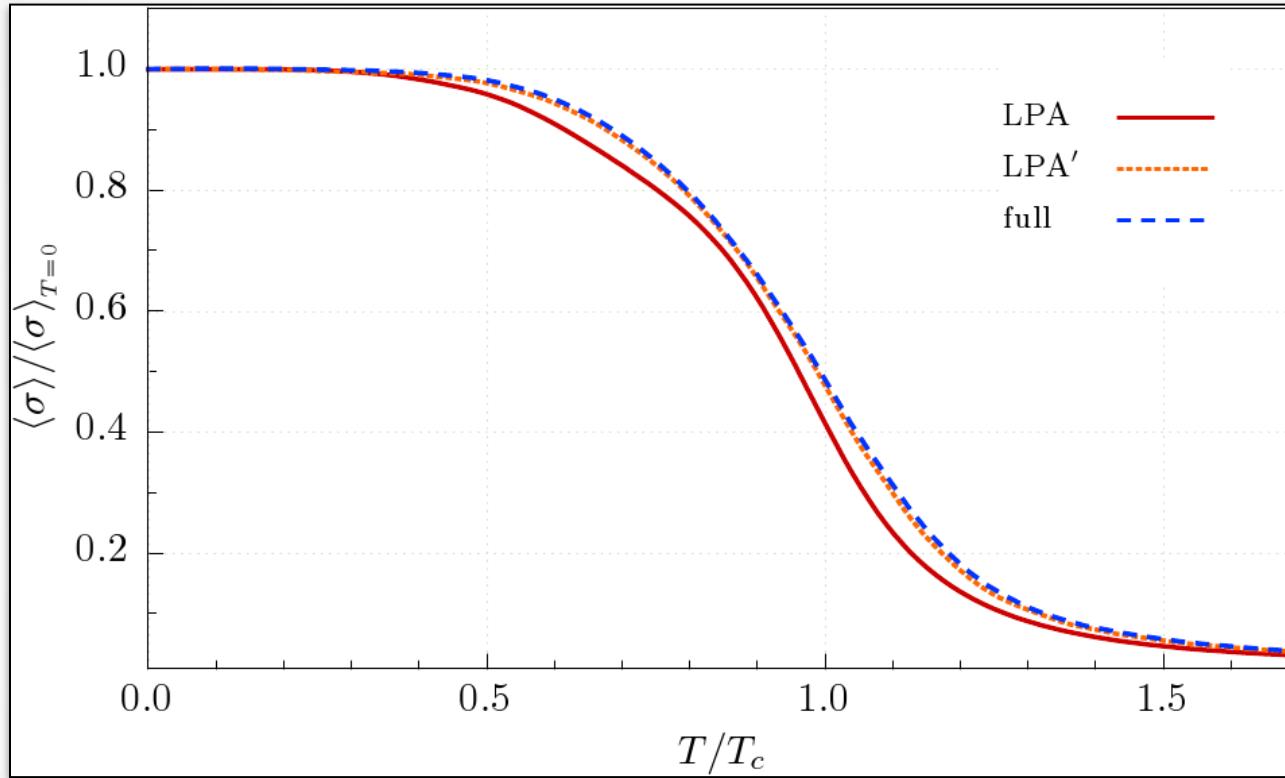
LPA' Comparison



- LPA' includes only a **scale-dependent Z**
- Very good approximation to the full calculation (**deviation < 3 %**)
- Upgrade: calculate momentum dependence on LPA' solution (1 step)

Comparison: Fixed IR

Model perspective



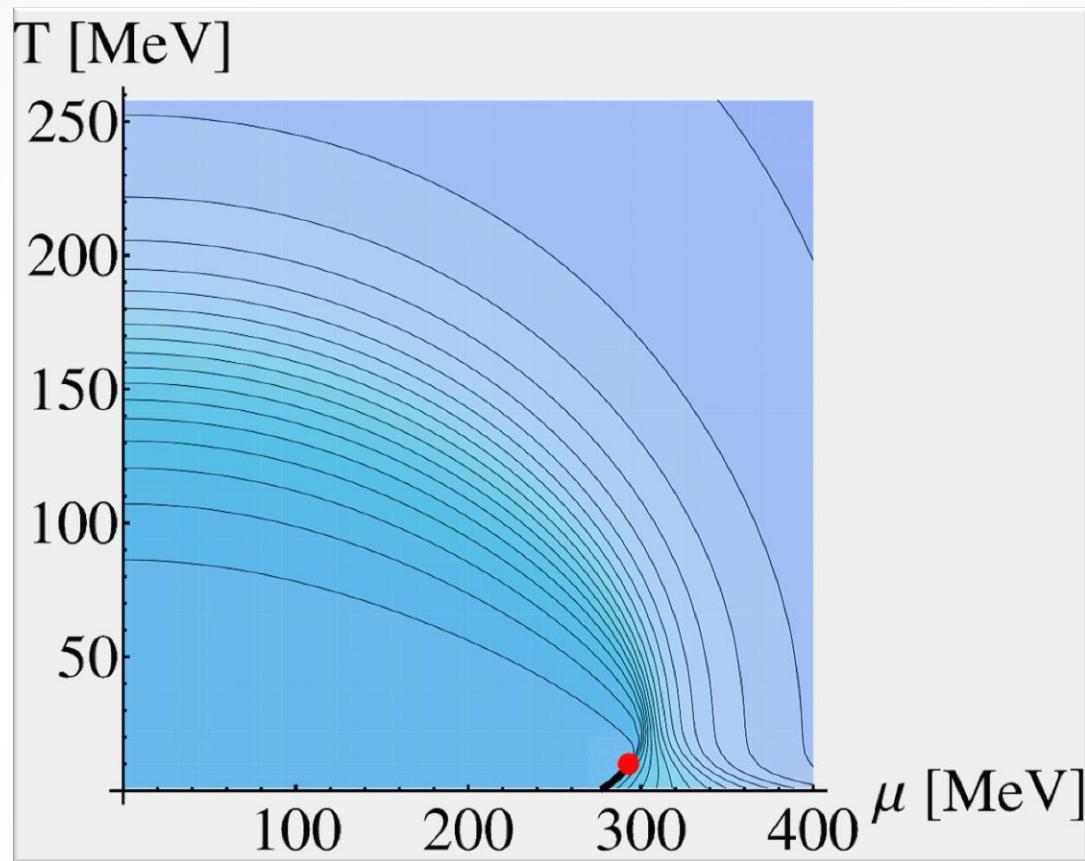
- Reasonably good agreement at $\mu=0$ (in terms of relative scales)
- But in LPA still **large systematic error at finite μ**

Spectral Functions

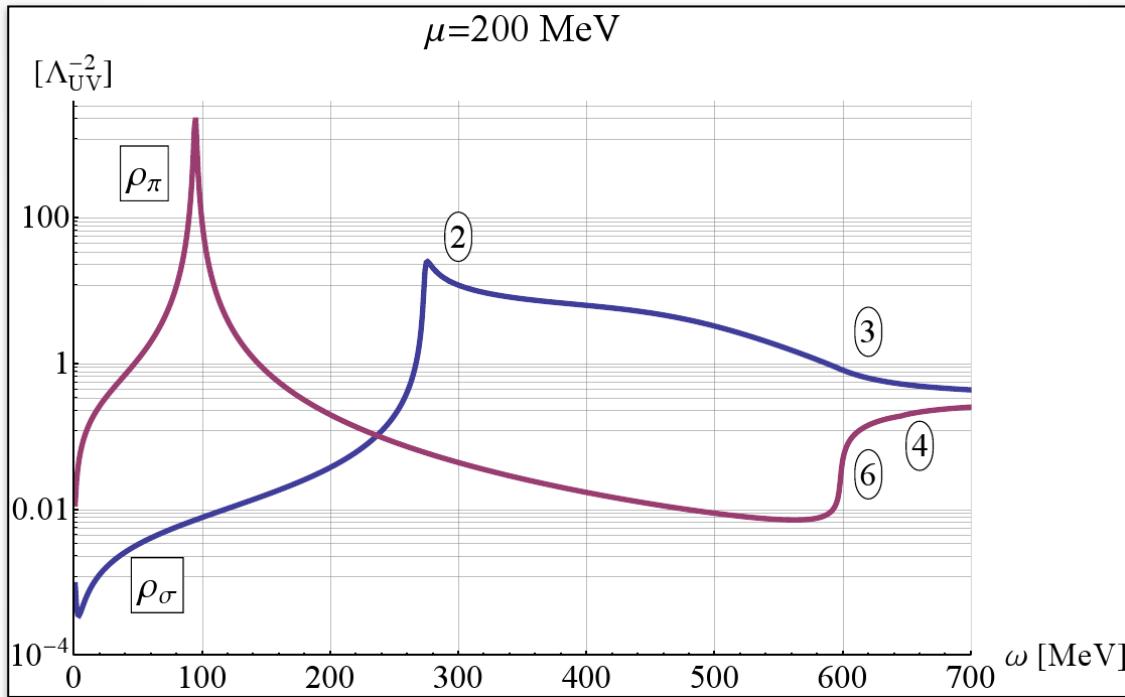
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- Kamikado, NSt, von Smekal, Wambach; Eur.Phys.J. **C74** (2014) 2806
- Tripolt, NSt, von Smekal, Wambach; Phys.Rev. **D89** (2014) 034010

QM Model at $\mu > 0$

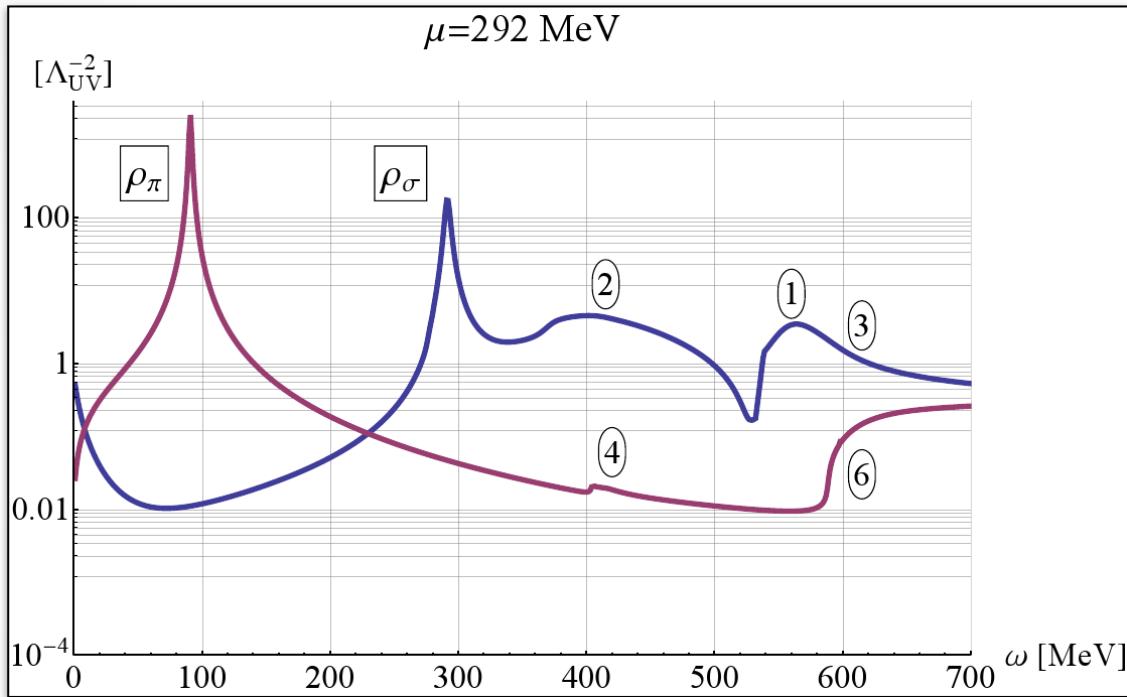


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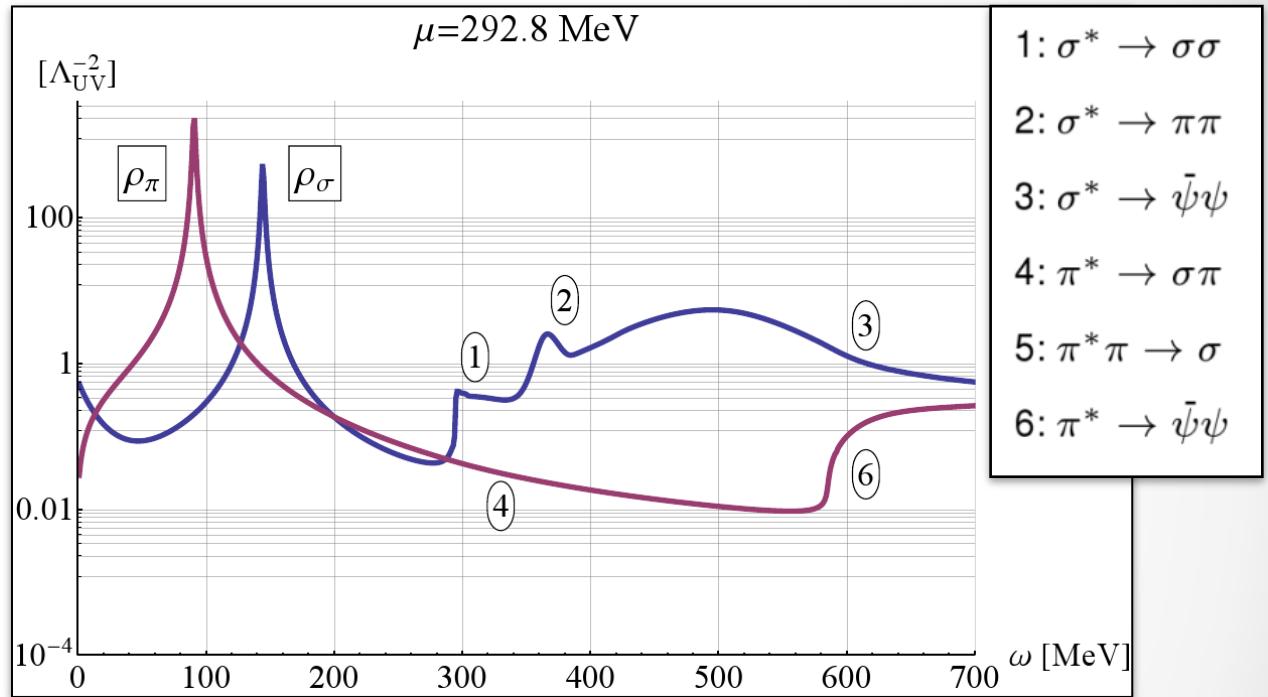
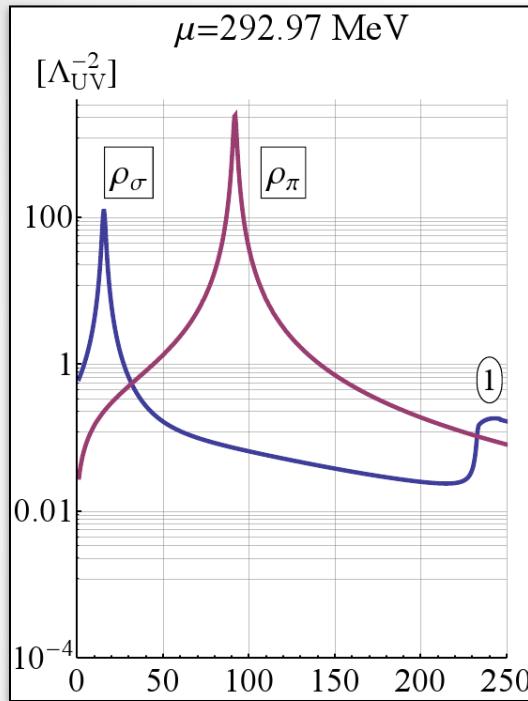
- 1: $\sigma^* \rightarrow \sigma\sigma$
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- 6: $\pi^* \rightarrow \bar{\psi}\psi$

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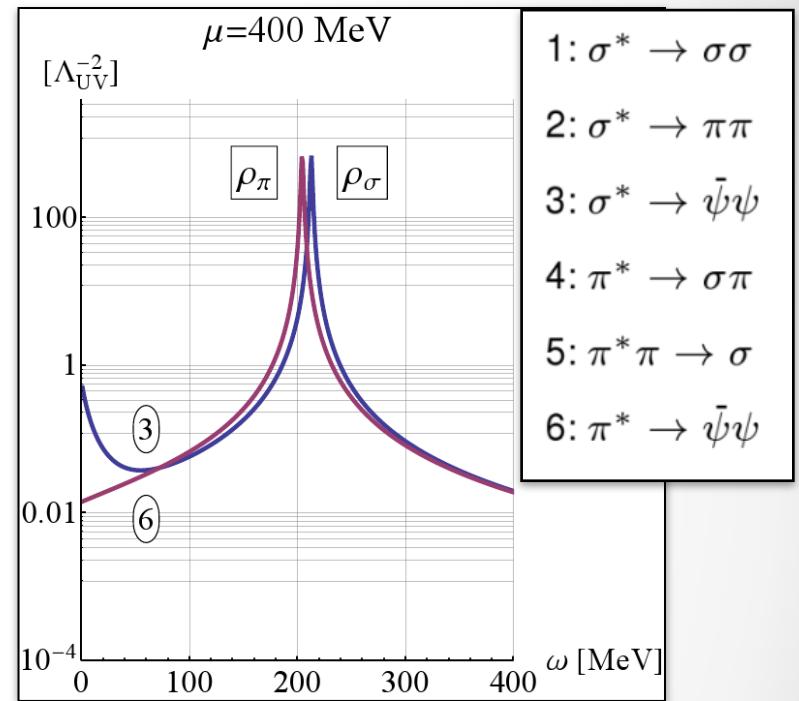


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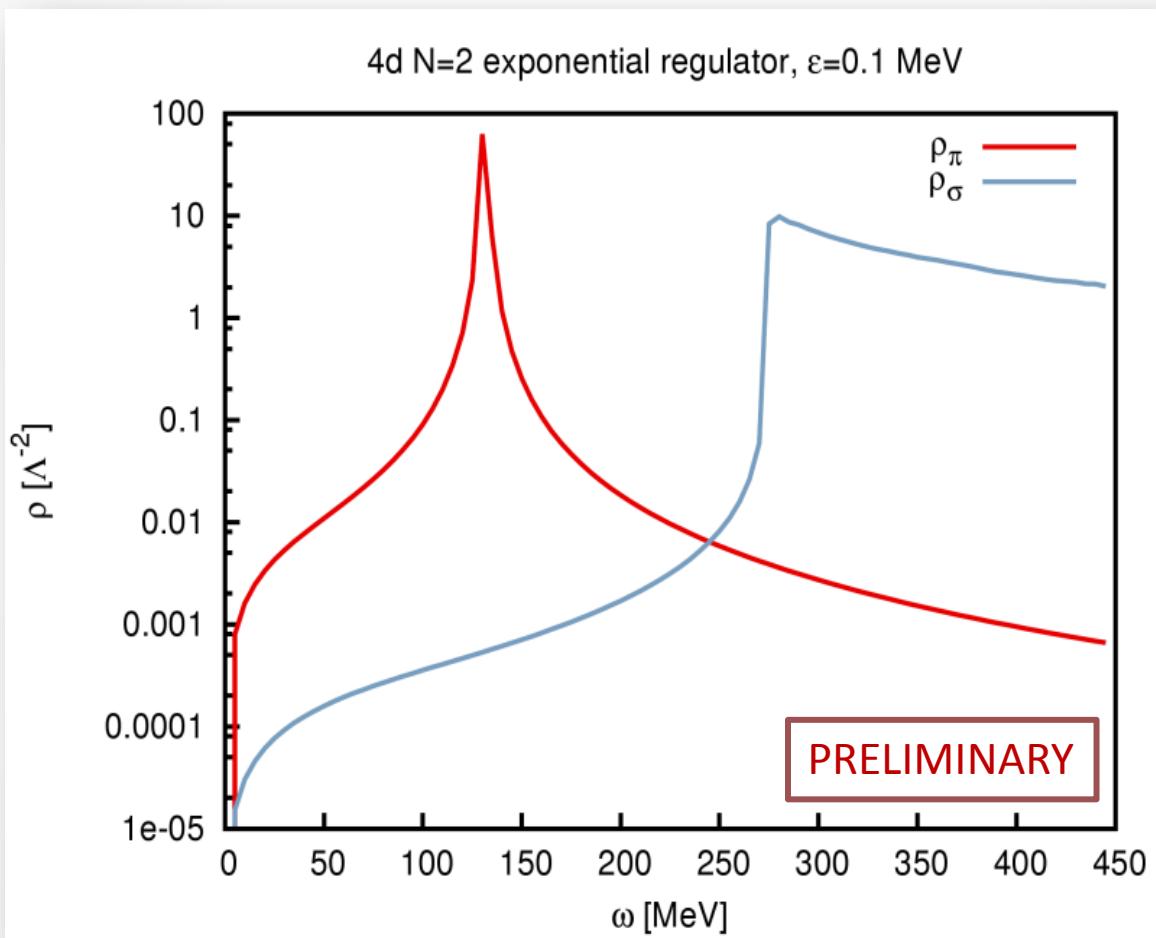
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Shopping list:

- Proper regulator for complex external momenta
- Suitable for numerical applications
- Analytical functions
- Analytical structure of regularized propagator: as few poles as possible
 - Require pole procedures to obtain the correct real-time result

4d Spectral Functions



➤ Pawłowski, NSt [in prep.]